Lesson 24: The Volume of a Right Prism

Student Outcomes

- Students use the formula for the volume of a right rectangular prism to answer questions about the capacity of tanks.
- Students compute volumes of right prisms involving fractional values for length.

Related Topics: More Lesson Plans for Grade 7 Common Core Math

Lesson Notes

Students extend their knowledge about the volume of solid figures to the notion of liquid volume. The Opening Exercise for Lesson 24 requires a small amount of water. Have an absorbent towel available to soak up the water at the completion of the exercise.

Classwork

Opening Exercise (3 minutes)

Pour enough water onto a large flat surface to form a puddle. Have students discuss how to determine the volume of the water. Provide 2 minutes for student discussion, and then start the class discussion.

Discussion (3 minutes)

- Why can't we easily determine the volume of the water in the puddle?
  - *The puddle does not have any definite shape or depth that we can easily measure.*
- How can we measure the volume of the water in three dimensions?
  - *The volume can be measured in three dimensions if put into a container. In a container, such as a prism, water takes on the shape of the container. We can measure the dimensions of the container to determine an approximate volume of the water in cubic units.*

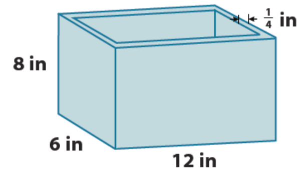
Example 1 (8 minutes): Measuring a Container's Capacity

Students progress from measuring the volume of a liquid inside a right rectangular prism filled to capacity to solving a variety of problems involving liquids and prism-shaped containers.

Ask questions to guide students in discovering the need to account for the thickness of the container material in determining the "inside" volume of the container. For instance, ask "Is the length of the inside of the container 12 inches? Why not? What is the width of the inside container? The depth? Why did you have to subtract twice the thickness to get the length and width, but only one times the thickness to get the depth?

Example 1: Measuring a Container's Capacity

A box in the shape of a right rectangular prism has a length of 12 in., a width of 6 in., and a height of 8 in. The base and the walls of the container are $\frac{1}{4}$ in. thick, and its top is open. What is the capacity of the right rectangular prism? (Hint: The capacity is equal to the volume of water needed to fill the prism to the top.)



If the prism is filled with water, the water will take the shape of a right rectangular prism slightly smaller than the container. The dimensions of the smaller prism are a length of $11\frac{1}{2}$ in, a width of $5\frac{1}{2}$ in, and a height of $7\frac{3}{4}$ in.

$V = Bh$

$V = (lw)h$

$V = \left(11\frac{1}{2}\ in \cdot 5\frac{1}{2}\ in\right) \cdot 7\frac{3}{4}\ in$

$V = \left(\frac{23}{2}\ in \cdot \frac{11}{2}\ in\right) \cdot \frac{31}{4} in$

$V = \left(\frac{253}{4} in^2\right) \cdot \frac{31}{4}\ in$

$V = \frac{7843}{16}\ in^3$

$V = 490\frac{3}{16}\ in^3$ *The capacity of the right rectangular prism is $490\frac{3}{16} in^3$.*

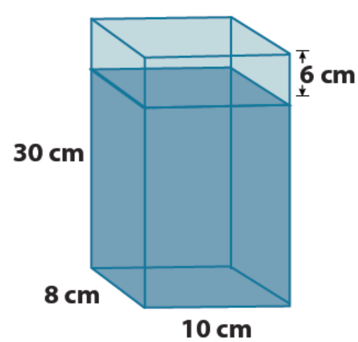
Example 2 (5 minutes): Measuring Liquid in a Container in Three Dimensions

Students use the inside of right prism-shaped containers to calculate the volumes of contained liquids.

Example 2: Measuring Liquid in a Container in Three Dimensions

A glass container is in the form of a right rectangular prism. The container is 10 cm long, 8 cm wide, and 30 cm high. The top of the container is open and the base and walls of the container are 3 mm (or 0.3 cm) thick. The water in the container is 6 cm from the top of the container. What is the volume of the water in the container?

Because of the walls and base of the container, the water in the container forms a right rectangular prism that is 9.4 cm long, 7.4 cm wide, and 23.7 cm tall.



$V = Bh$

$V = (lw)h$

$V = (9.4\ cm \cdot 7.4\ cm) \cdot 23.7 cm$

$V = \left(\frac{94}{10}\ cm \cdot \frac{74}{10}\ cm\right) \cdot \frac{237}{10} cm$

$V = \left(\frac{6956}{100}\ cm^2\right) \cdot \frac{237}{10} cm$

$V = \frac{1648572}{1000}\ cm^3$

$V = 1648.572\ cm^3$

The volume of the water in the container is $1648.572\ cm^3$.

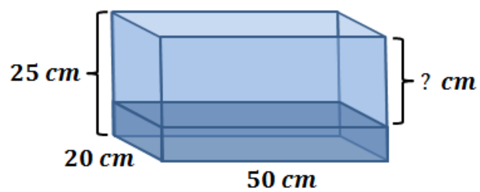
Example 3 (8 minutes)

Students determine the depth of a given volume of water in a container of given size.

Example 3

7.2 L of water are poured into a container in the shape of a right rectangular prism. The inside of the container is 50 cm long, 20 cm wide, and 25 cm tall. How far from the top of the container is the surface of the water? (1 L$=1000$ cm^3)

$7.2L = 7200\ cm^3$



$$V = Bh$$
$$V = (lw)h$$
$$7200\ cm^3 = (50\ cm)(20\ cm)h$$
$$7200\ cm^3 = 1000\ cm^2 \cdot h$$
$$7200\ cm^3 \cdot \frac{1}{1000\ cm^2} = 1000\ cm^2 \cdot \frac{1}{1000\ cm^2} \cdot h$$
$$\frac{7200}{1000}\ cm = 1 \cdot h$$
$$7.2\ cm = h$$

The depth of the water is $7.2\ cm$. The height of the container is $25\ cm$.

The surface of the water is $25\ cm - 7.2\ cm = 17.8 cm$ from the top of the container.

Example 4 (8 minutes)

Students find unknown measurements of a right prism given its volume and two dimensions.

Example 4

A fuel tank is the shape of a right rectangular prism and has 27 L of fuel in it. It is determined that the tank is $\frac{3}{4}$ full. The inside dimensions of the base of the tank are 90 cm by 50 cm. How deep is the fuel in the tank? How deep is the tank? (1 L$=1000$ cm^3)

Let the height of the tank be h cm.

$27\ L = 27000\ cm^3$

$$V = Bh$$
$$V = (lw)h$$
$$27000\ cm^3 = (90\ cm \cdot 50\ cm) \cdot h$$
$$27000\ cm^3 = (4500\ cm^2) \cdot h$$
$$27000\ cm^3 \cdot \frac{1}{4500\ cm^2} = 4500\ cm^2 \cdot \frac{1}{4500\ cm^2} \cdot h$$
$$\frac{27000}{4500}\ cm = 1 \cdot h$$
$$6\ cm = h$$

The depth of the fuel in the tank is $6\ cm$. The depth of the fuel is $\frac{3}{4}$ the depth of the tank. Let d represent the depth of the tank in in centimeters.

$$6\ cm = \frac{3}{4}d$$
$$6\ cm \cdot \frac{4}{3} = \frac{3}{4} \cdot \frac{4}{3} \cdot d$$

$8\ cm = d$ *The depth of the fuel tank is $8\ cm$.*

Closing (2 minutes)

- How do containers, such as prisms, allow us to measure the volumes of liquids using three dimensions?
  - *When liquid is poured into a container, the liquid takes on the shape of the container's interior. We can measure the volume of prisms in three dimensions, allowing us to measure the volume of the liquid in three dimensions.*
- What special considerations have to be made when measuring liquids in containers in three dimensions?
  - *The outside and inside dimensions of a container will not be the same because the container has wall thickness. In addition, whether or not the container is filled to capacity will affect the volume of the liquid in the container.*

Exit Ticket (8 minutes)

Students may be allowed to use calculators when completing this Exit Ticket.

Name ____ Date ____________________

Lesson 24: The Volume of a Right Prism

Exit Ticket

1. Lawrence poured 27.328 liters of water into a right rectangular prism-shaped tank. The base of the tank is 40 cm by 28 cm. When he finished pouring the water, the tank was $\frac{2}{3}$ full. (1 liter$=$ 1000 cm^3)

   a. How deep is the water in the tank?

   b. How deep is the tank?

   c. How many liters of water can the tank hold in total?

Exit Ticket Sample Solutions

1. Lawrence poured 27.328 liters of water into a right rectangular prism shaped tank. The base of the tank is 40 cm by 28 cm. When he finished pouring the water, the tank was $\frac{2}{3}$ full. (1 liter$=1000$ cm^3)

   a. How deep is the water in the tank?

   $27.328\ L = 27328\ cm^3$

   $V = Bh$

   $V = (lw)h$

   $27328\ cm^3 = (40cm \cdot 28cm) \cdot h$

   $27328\ cm^3 = 1120\ cm^2 \cdot h$

   $27328\ cm^3 \cdot \frac{1}{1120cm^2} = 1120\ cm^2 \cdot \frac{1}{1120cm^2} \cdot h$

   $\frac{27328}{1120} cm = 1 \cdot h$

   $24\frac{280}{1120} cm = h$

   $24\frac{2}{5} cm = h$ *The depth of the water is $24\frac{2}{5}$ cm.*

   b. How deep is the tank?

   The depth of the water is $\frac{2}{3}$ the depth of the tank. Let d represent the depth of the tank in centimeters.

   $24\frac{2}{5}\ cm = \frac{2}{3} \cdot d$

   $24\frac{2}{5}\ cm \cdot \frac{3}{2} = \frac{2}{3} \cdot \frac{3}{2} \cdot d$

   $36\ cm + \frac{3}{5}\ cm = 1d$

   $36\frac{3}{5}\ cm = d$ *The depth of the tank is $36\frac{3}{5}$ cm.*

   c. How many liters of water can the tank hold total?

   $V = Bh$

   $V = (lw)h$

   $V = (40\ cm \cdot 28\ cm) \cdot 36\frac{3}{5}\ cm$

   $V = 1120\ cm^2 \cdot 36\frac{3}{5} cm$

   $V = 40320\ cm^3 + 672\ cm^3$

   $V = 40992\ cm^3$

   $40992\ cm^3 = 40.992\ L$ *The tank can hold up to 40.992 L of water.*

Problem Set Sample Solutions

1. **Mark wants to put some fish and some decorative rocks in his new glass fish tank. He measured the outside dimensions of the right rectangular prism and recorded a length of 55 cm, width of 42 cm, and height of 38 cm. He calculates that the tank will hold 87.78 L of water. Why is Mark's calculation of volume incorrect? What is the correct volume? Mark also failed to take into account the fish and decorative rocks he plans to add. How will this affect the volume of water in the tank? Explain.**

   $V = Bh = (lw)h$

   $V = 55\ cm \cdot 42\ cm \cdot 38\ cm$

   $V = 2310\ cm^2 \cdot 38\ cm$

   $V = 87780\ cm^3$

   $87780\ cm^3 = 87.78\ L$

   Mark measured only the outside dimensions of the fish tank and did not account for the thickness of the sides of the tank. If he fills the tank with $87.78\ L$ of water, the water will overflow the sides. Mark also plans to put fish and rocks in the tank which will force water out of the tank if it is filled to capacity.

2. **Leondra bought an aquarium that is a right rectangular prism. The inside dimensions of the aquarium are 90 cm long, by 48 cm wide, by 60 cm deep. She plans to put water in the aquarium before purchasing any pet fish. How many liters of water does she need to put in the aquarium so that the water level is 5 cm below the top?**

   If the aquarium is 60 cm deep, then she wants the water to be 55 cm deep. Water takes on the shape of its container so the water will form a right rectangular prism with a length of 90 cm, a width of 48 cm, and a height of 55 cm.

   $V = Bh = (lw)h$

   $V = (90\ cm \cdot 48cm) \cdot 55\ cm$

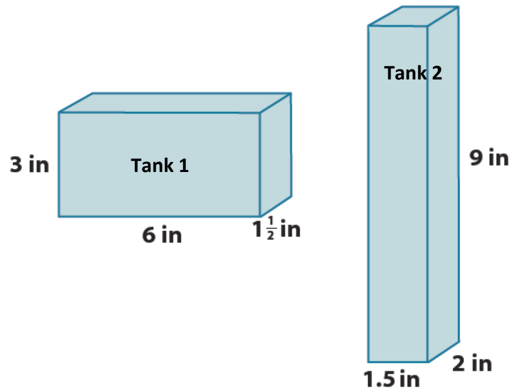
   $V = 4320\ cm^2 \cdot 55\ cm$

   $V = 237600\ cm^3$

   $237600\ cm^3 = 237.6\ L$

   The volume of water needed is $237.6\ L$.

3. The inside space of two different water tanks are shown below. Which tank has a greater capacity? Justify your answer.



$V_1 = Bh = (lw)h$

$V_1 = \left(6\ in \cdot 1\frac{1}{2}\ in\right) \cdot 3\ in$

$V_1 = (6\ in^2 + 3\ in^2) \cdot 3\ in^2$

$V_1 = 9\ in^2 \cdot 3\ in^2$

$V_1 = 27\ in^3$

$V_2 = Bh = (lw)h$

$V_2 = \left(1\frac{1}{2} in \cdot 2\ in\right) \cdot 9\ in$

$V_2 = (2\ in^2 + 1 in^2) \cdot 9\ in$

$V_2 = 3\ in^2 \cdot 9\ in$

$V_2 = 27\ in^3$

The tanks have the same volume, $27\ in^3$. Each prism has a face with an area of $18\ in^2$ (base) and a height that is $1\frac{1}{2} in$.

4. The inside of a tank is in the shape of a right rectangular prism. The base of that prism is 85 cm by 64 cm. What is the minimum height inside the tank if the volume of the liquid in the tank is 92 L?

$V = Bh = (lw)h$

$92000\ cm^3 = (85\ cm \cdot 64\ cm) \cdot h$

$92000\ cm^3 = 5440\ cm^2 \cdot h$

$92000\ cm^3 \cdot \frac{1}{5440\ cm^2} = 5440\ cm^2 \cdot \frac{1}{5440\ cm^2} \cdot h$

$\frac{92000}{5440} cm = 1 \cdot h$

$16\frac{31}{34} cm = h$

The minimum height of the inside of the tank is $16\frac{31}{34}$ cm.

5. An oil tank is the shape of a right rectangular prism. The inside of the tank is 36.5 cm long, 52 cm wide, and 29 cm high. If 45 liters of oil have been removed from the tank since it was full, what is the current depth of oil left in the tank?

$V = Bh = (lw)h$

$V = (36.5\ cm \cdot 52\ cm) \cdot 29cm$

$V = 1898\ cm^2 \cdot 29\ cm$

$V = 55042\ cm^3$

The tank has a capacity of $55042\ cm^3$, or $55.042\ L$. If $45\ L$ of oil have been removed from the tank, then

$55.042\ L - 45\ L = 10.042\ L$ are left in the tank.

$$V = Bh = (lw)h$$
$$10042\ cm^3 = (36.5\ cm \cdot 52\ cm) \cdot h$$
$$10042\ cm^3 = 1898\ cm^2 \cdot h$$
$$10042\ cm^3 \cdot \frac{1}{1898\ cm^2} = 1898\ cm^2 \cdot \frac{1}{1898\ cm^2} \cdot h$$
$$\frac{10042}{1898}\ cm = 1 \cdot h$$
$$5.29\ cm \approx h$$

The depth of oil left in the tank is approximately $5.29\ cm$.

6. The inside of a right rectangular prism-shaped tank has a base that is 14 cm by $24\ cm$ and a height of 60 cm. The tank is filled to its capacity with water, then 10.92 L of water is removed. How far did the water level drop?

$V = Bh = (lw)h$

$V = (14\ cm \cdot 24\ cm) \cdot 60cm$

$V = 336\ cm^2 \cdot 60\ cm$

$V = 20160\ cm^3$

The capacity of the tank is $20160\ cm^3$ or $20.16\ L$. When $10.92\ L$ or $10920\ cm^3$ of water is removed from the tank, there remains $20160\ cm^3 - 10920\ cm^3 = 9240\ cm^3$ of water in the tank.

$$V = Bh = (lw)h$$
$$9240\ cm^3 = (14\ cm \cdot 24\ cm) \cdot h$$
$$9240\ cm^3 = 336\ cm^2 \cdot h$$
$$9240\ cm^3 \cdot \frac{1}{336\ cm^2} = 336\ cm^2 \cdot \frac{1}{336\ cm^2} \cdot h$$
$$\frac{9240}{336}\ cm = 1 \cdot h$$
$$27\frac{1}{2}\ cm = h$$

The depth of the water left in the tank is $27\frac{1}{2}$ cm. This means that the water level has dropped

$60cm - 27\frac{1}{2}\ cm = 32\frac{1}{2}\ cm$.

7. **A right rectangular prism-shaped container has inside dimensions of $7\frac{1}{2}$ cm long and $4\frac{3}{5}$ cm wide. The tank is $\frac{3}{5}$ full of vegetable oil. It contains 0.414 liters of oil. Find the height of the container.**

$V = Bh = (lw)h$

$414\ cm^3 = \left(7\frac{1}{2} cm \cdot 4\frac{3}{5} cm\right) \cdot h$

$414\ cm^3 = 34\frac{1}{2}\ cm^2 \cdot h$

$414\ cm^3 = \frac{69}{2} cm^2 \cdot h$

$414\ cm^3 \cdot \frac{2}{69cm^2} = \frac{69}{2} cm^2 \cdot \frac{2}{69cm^2} \cdot h$

$\frac{828}{69}\ cm = 1 \cdot h$

$12\ cm = h$ *The vegetable oil in the container is 12 cm deep, but this is only $\frac{3}{5}$ of the containers depth. Let d represent the depth of the container in centimeters.*

$12\ cm = \frac{3}{5} \cdot d$

$12\ cm \cdot \frac{5}{3} = \frac{3}{5} \cdot \frac{5}{3} \cdot d$

$\frac{60}{3} cm = 1 \cdot d$

$20\ cm = d$ *The depth of the container is 20 cm.*

8. A right rectangular prism with length of 10 in., width of 16 in., and height of 12 in. is $\frac{2}{3}$ filled with water. If the water is emptied into another right rectangular prism with a length of 12 in., a width of 12 in., and height of 9 in., will the second container hold all the water? Explain why or why not. Determine how far (above or below) the water level would be from the top of the container.

$\frac{2}{3} \cdot 12\ in = \frac{24}{3}\ in = 8\ in$ *The height of the water in the first prism is $8\ in$.*

$V = Bh = (lw)h$

$V = (10\ in \cdot 16\ in) \cdot 8\ in$

$V = 160\ in^2 \cdot 8\ in$

$V = 1280\ in^3$ *The volume of water is $1280\ in^3$.*

$V = Bh = (lw)h$

$V = (12\ in \cdot 12\ in) \cdot 9\ in$

$V = 144\ in^2 \cdot 9\ in$

$V = 1296\ in^3$ *The capacity of the second prism is $1296\ in^3$, which is greater than the volume of water, so the water will fit in the second prism.*

$V = Bh = (lw)h$ *Let h represent the depth of the water in the second prism in inches.*

$1280\ in^3 = (12\ in \cdot 12\ in) \cdot h$

$1280\ in^3 = (144\ in^2) \cdot h$

$1280\ in^3 \cdot \frac{1}{144 in^2} = 144\ in^2 \cdot \frac{1}{144 in^2} \cdot h$

$\frac{1280}{144}\ in = 1 \cdot h$

$8\frac{128}{144}\ in = h$

$8\frac{8}{9}\ in = h$ *The depth of the water in the second prism is $8\frac{8}{9}\ in$.*

The water level will be $9\ in - 8\frac{8}{9}\ in = \frac{1}{9}\ in$ from the top of the second prism.