



Student Outcomes

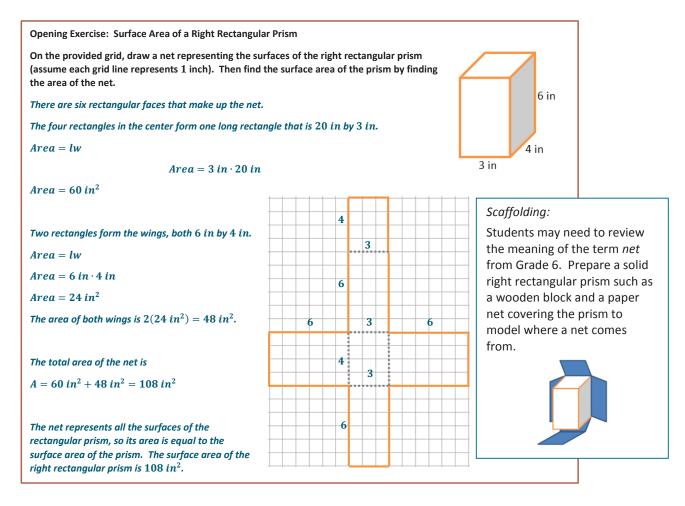
 Students find the surface area of three-dimensional objects whose surface area is composed of triangles and quadrilaterals. They use polyhedron nets to understand that surface area is simply the sum of the area of the lateral faces and the area of the base(s).

Related Topics: More Lesson Plans for Grade 7 Common Core Math

Classwork

Opening Exercise (8 minutes): Surface Area of a Right Rectangular Prism

Students use prior knowledge to find the surface area of the given right rectangular prism by decomposing the prism into the plane figures that represent its individual faces. Students then discuss their methods aloud.



Note to teacher: Students may draw any of the variations of nets for the given prism.



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Discussion (3 minutes)

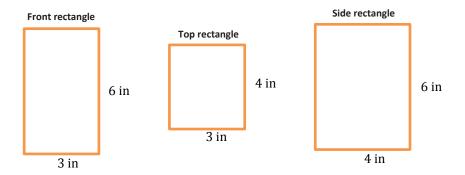
- What other ways could we have found the surface area of the rectangular prism?
 - Surface area formula: SA = 2lw + 2lh + 2wh

$$SA = 2(3 \text{ in} \cdot 4 \text{ in}) + 2(3 \text{ in} \cdot 6 \text{ in}) + 2(4 \text{ in} \cdot 6 \text{ in})$$

 $SA = 24 \text{ in}^2 + 36 \text{ in}^2 + 48 \text{ in}^2$
 $SA = 108 \text{ in}^2$

Lesson 21

• Find the areas of each individual rectangular face:



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$$Area = length \times width$$

 $A = 6 \text{ in } \times 3 \text{ in} \qquad A = 4 \text{ in } \times 3 \text{ in} \qquad A = 6 \text{ in } \times 4 \text{ in}$ $A = 18 \text{ in}^2 \qquad A = 12 \text{ in}^2 \qquad A = 24 \text{ in}^2$ There are two of each face, so $SA = 2(18 \text{ in}^2 + 12 \text{ in}^2 + 24 \text{ in}^2)$ $SA = 2(54 \text{ in}^2)$ $SA = 108 \text{ in}^2$

Discussion (6 minutes): Terminology

A right prism can be described as a solid with two "end" faces (called its *bases*) that are exact copies of each other and rectangular faces that join corresponding edges of the bases (called *lateral faces*).

- Are the bottom and top faces of a right rectangular prism the bases of the prism?
 - Not always. Any of its opposite faces can be considered bases because they are all rectangles.
- If we slice the right rectangular prism in half along a diagonal of a base (see picture), the two halves are called right triangular prisms. Why do you think they are called triangular prisms?
 - The bases of each prism are triangles, and prisms are named by their bases.
- Why must the triangular faces be the bases of these prisms?
 - Because the lateral faces (faces that are not bases) of a right prism have to be rectangles.
- Can the surface area formula for a right rectangular prism (SA = 2lw + 2lh + 2wh) be applied to find the surface area of a right triangular prism? Why or why not?
 - No, because each of the terms in the surface area formula represents the area of a rectangular face. A right triangular prism has bases that are triangular, not rectangular.





5 in

3 in

6 in

4 in





Exercise 1 (8 minutes)

Students find the surface area of the right triangular prism to determine the validity of a given conjecture.

Exercise 1

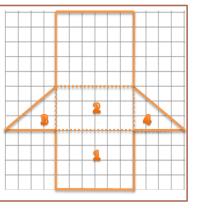
Marcus thinks that the surface area of the right triangular prism will be half that of the right rectangular prism and wants to use the modified formula $SA = \frac{1}{2}(2lw + 2lh + 2wh)$. Do you agree or disagree with Marcus? Use nets of the prisms to support your argument.

The surface area of the right rectangular prism is 108 in², so Marcus believes the surface areas of each right triangular prism is 54 in^2 .

Students can make comparisons of the area values depicted in the nets of the prisms and can also compare the physical areas of the nets either by overlapping the nets on the same grid or using a transparent overlay.

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The net of the right triangular prism has one less face than the right rectangular prism. Two of the rectangular faces on the right triangular prism (rectangular regions 1 and 2 in the diagram) are the same faces from the right rectangular prism, so they are the same size. The areas of the triangular bases (triangular regions 3 and 4 in the diagram) are half the area of their corresponding rectangular faces of the right rectangular prism. These four faces of the right triangular prism make up half the surface area of the right rectangular prism before considering the fifth face, so no, Marcus is incorrect.



The areas of rectangular faces 1 and 2, plus the areas of the triangular regions 3 and 4 is 54 in^2 . The last rectangular region has an area of 30 in^2 . The total area of the net is $54 + 30 = 84 in^2$, which is far more than half the surface area of the right rectangular prism.

Use a transparency to show students how the nets overlap where the lateral faces together form a longer rectangular region and the bases are represented by "wings" on either side of that triangle. You may want to use student work for this if you see a good example. Use this setup in the following discussion.

Discussion (5 minutes)

- The surface area formula (SA = 2lw + 2lh + 2wh) for a right rectangular prism cannot be applied to a right triangular prism. Why?
 - The formula adds the areas of six rectangular faces. A right triangular prism only has four rectangular faces and also has two triangular faces (bases).
- The area formula for triangles is $\frac{1}{2}$ the formula for the area of rectangles or parallelograms. Can the surface area of a triangular prism be obtained by dividing the surface area formula for a right rectangular prism by 2? Explain.
 - No. The right triangular prism in the above example had more than half the surface area of the right rectangular prism that it was cut from. If this occurs in one case, then it must occur in others as well.



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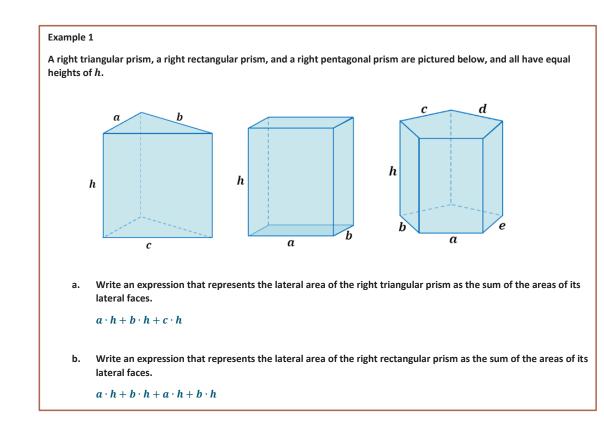
- If you compare the nets of the right rectangular prism and the right triangular prism, what do the nets seem to have in common? (Hint: What do all right prisms have in common? Answer: Rectangular lateral faces.)
 - Their lateral faces form a larger rectangular region and the bases are attached to the side of that rectangle like "wings".
- Will this commonality always exist in right prisms? How do you know?
 - Yes! Right prisms must have rectangular lateral faces. If we align all the lateral faces of a right prism in a net, they can always form a larger rectangular region because they all have the same height as the prism.
- How do we determine the total surface area of the prism?
 - Add the total area of the lateral faces and the areas of the bases
 If we let LA represent the lateral area and let B represent the area of a base, then the surface area of a right prism can be found using the formula:
 SA = LA + 2B

Scaffolding:

The teacher may need to assist students at finding the commonality between the nets of right prisms by showing examples of various right prisms and the fact that they all have rectangular lateral faces. The rectangular faces may be described as "connectors" between the bases of a right prism.

Example 1 (6 minutes): Lateral Area of a Right Prism

Students find the lateral areas of right prisms and recognize the pattern of multiplying the height of the right prism (the distance between its bases) by the perimeter of the prism's base.





MP.8

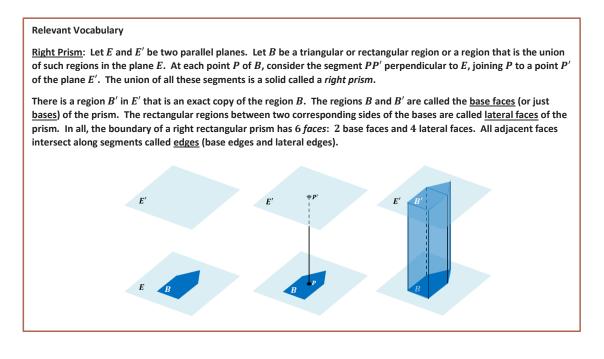
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	c.	Write an expression t lateral faces.	hat represents the lateral area of	the right pentagonal prism as the sum	of the areas of its	
		$a \cdot h + b \cdot h + c \cdot h + c$	$d\cdot h + e\cdot h$			
	d.	What value appears o	often in each expression and why	?	Scaffolding:	
		h; Each prism has a h	eight of h, and therefore each la	teral face has a height of .	Example 1 can further by assi	be explored gning numbers
	e.	Rewrite each expressi each lateral face.	on in factored form using the dis	tributive property and the height of	to represent the sides of the ba	he lengths of the ases of each
P.8		h(a+b+c)	h(a+b+a+b)	h(a+b+c+d+e)	· ·	ents represent a as the sum of
	f.	perimeter	eses in each case represent with r $h(\overline{a+b+a+b})$	perimeter	without evalua	ne lateral faces ating, the or in each term
		h(a+b+c) The perimeter of the b	n(a + b + a + b) base of the corresponding prism.	n(a+b+c+a+e)	factored out to	t and can then be o reveal the same
	g.	How can we generaliz right prisms?	e the lateral area of a right prism	into a formula that applies to all	relationship.	
			ateral area of a right prism, P rep ce between the right prism's base	resents the perimeter of the right prisi s, then:	m's base, and h	
		$LA = P_{base} \cdot h$				

Closing (5 minutes)

The vocabulary below contains the precise definitions of the visual/colloquial descriptions used in the lesson. Please read through the definitions aloud with your students, asking them questions that compare the visual/colloquial descriptions used in the lesson with the precise definitions.





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Surface Area



<u>Cı</u>	ube: A cube is a right rectangular prism all of whose edges are of equal length.
<u>Sı</u>	<u>urface</u> : The surface of a prism is the union of all of its faces (the base faces and lateral faces).
N	et (description): A net is a two dimensional diagram of the surface of a prism.
1.	Why are the lateral faces of right prisms always rectangular regions?
	Because along a base edge, the line segments PP' are always perpendicular to the edge, forming a rectangular region.
2.	What is the name of the right prism whose bases are rectangles?
	Right rectangular prism.
3.	How does this definition of right prism include the interior of the prism?
	The union of all the line segments fills out the interior.
Г	
	Lesson Summary
	The surface area of a right prism can be obtained by adding the areas of the lateral faces to the area of the bases. The formula for the surface area of a right prism is $SA = LA + 2B$, where SA represents surface area of the prism, LA represents the area of the lateral faces, and B represents the area of one base. The lateral area LA can be obtained by multiplying the perimeter of the base of the prism times the height of the prism.

Exit Ticket (4 minutes)







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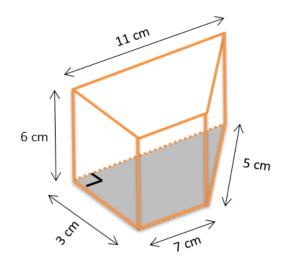
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Lesson 21: Surface Area

Exit Ticket

Find the surface area of the right trapezoidal prism. Show all necessary work.









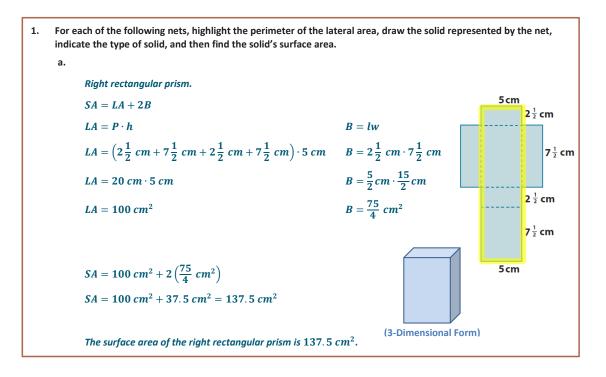
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Exit Ticket Sample Solutions

Find the surface area of the right trapezoidal prism. Show all necessary work. 11 cm SA = LA + 2B $LA = P \cdot h$ $LA = (3 + 7 + 5 + 11) cm \cdot 6 cm$ $LA = 26 \ cm \cdot 6 \ cm$ $LA = 156 \ cm^2$ 6 cm Each base consists of a 3 cm by 7 cm rectangle and right triangle with a 5 cm base of 3 cm and a height of 4 $cm.\,$ Therefore, the area of each base: $B = A_r + A_t$ ્રર્જ $B = lw + \frac{1}{2}bh$ K 7 cm $B = (7 \ cm \cdot 3 \ cm) + \left(\frac{1}{2} \cdot 3 \ cm \cdot 4 \ cm\right)$ $B=21\,cm^2+6\,cm^2$ $B = 27 \ cm^2$ SA = LA + 2B $SA = 156 \ cm^2 + 2(27 \ cm^2)$ $SA = 156 \ cm^2 + 54 \ cm^2$ $SA = 210 \ cm^2$ The surface of the right trapezoidal prism is $210 \ cm^2$.

Problem Set Sample Solutions





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b.

2.

a.

b.

SA = LA + 2B $LA = P \cdot h$ $B=\frac{1}{2}bh$ 10 in $B=\frac{1}{2}(8\,in)\left(9\frac{1}{5}\,in\right)$ $LA = (10 in + 8 in + 10 in) \cdot 12 in$ $B=4in\left(9\frac{1}{5}in\right)$ 9 ½ in $LA = 28 in \cdot 12 in$ 8 in $B = \left(36 + \frac{4}{5}\right)in^2 = 36\frac{4}{5}in^2$ $LA = 336 in^2$ 10 in $SA = 336 in^2 + 2(36\frac{4}{5}in^2)$ 12 in $SA = 336 in^2 + (72 + \frac{8}{5})in^2$ $SA = 408 \ in^2 + 1\frac{3}{5} \ in^2$ $SA = 409\frac{3}{5} in^2$ The surface area of the right triangular prism is $409\frac{3}{5}$ in². (3-Dimensional Form) Given a cube with edges that are $\frac{3}{4}$ inch long: Find the surface area of the cube. $SA = 6s^2$ $SA = 6\left(\frac{3}{4}in\right)^2$ $SA = 6\left(\frac{3}{4}in\right) \cdot \left(\frac{3}{4}in\right)$ $SA = 6\left(\frac{9}{16}in^{2}\right)$ $SA = \frac{27}{8}in^{2} = 3\frac{3}{8}in^{2}$ Joshua makes a scale drawing of the cube using a scale factor of 4. Find the surface area of the cube that Joshua drew.

 $\frac{3}{4}$ in $\cdot 4 = 3$ in; The edge lengths of Joshua's drawing would be 3 inches.

 $SA = 6(3 in)^2$ $SA = 6(9 in^2) = 54 in^2$

c. What is the ratio of the surface area of the scale drawing to the surface area of the actual cube, and how does the value of the ratio compare to the scale factor?

 $54 \cdot \frac{8}{27}$ $2 \cdot 8 = 16$. The ratios of the surface area of the scale drawing to the surface area of the actual cube is 16:1. The value of the ratio is 16. The scale factor of the drawing is 4, and the value of the ratio of the surface area of the drawing to the surface area of the actual cube is $4^2 = 16$.

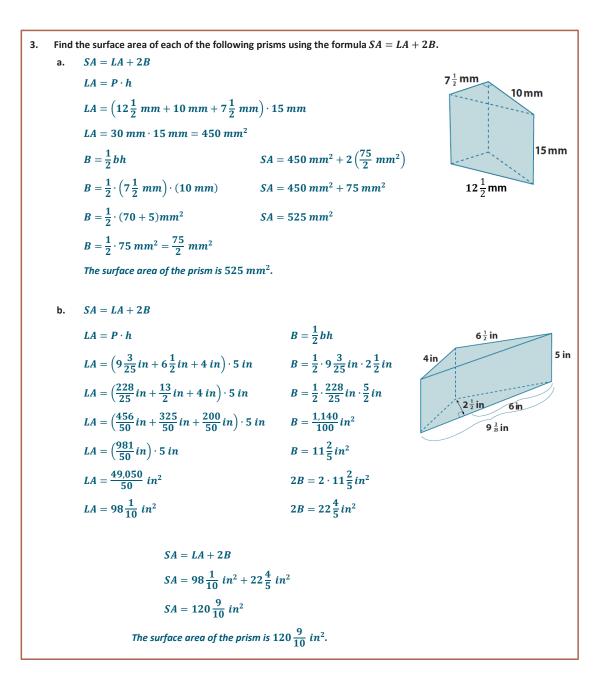




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 $54 \div 3\frac{3}{8}$

 $54 \div \frac{27}{9}$



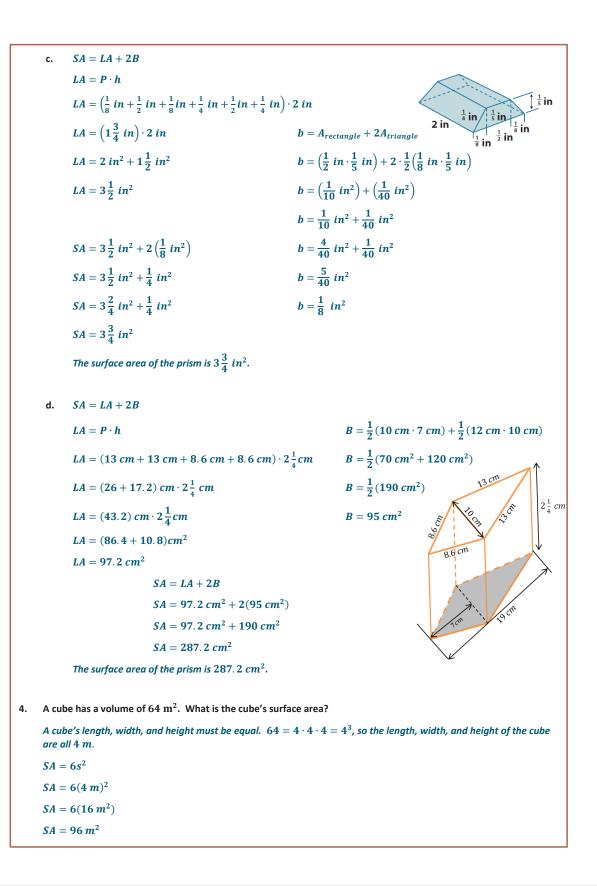


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5. The height of a right rectangular prism is $4\frac{1}{2}$ ft. The length and width of the prism's base are 2 ft and $1\frac{1}{2}$ ft. Use the formula SA = LA + 2B to find the surface area of the right rectangular prism. SA = LA + 2B $LA = P \cdot h$ b = lwSA = LA + 2b $LA = \left(2 ft + 2 ft + 1\frac{1}{2} ft + 1\frac{1}{2} ft\right) \cdot 4\frac{1}{2} ft \qquad b = 2 ft \cdot 1\frac{1}{2} ft \qquad SA = 31\frac{1}{2} ft^2 + 2(3 ft^2)$ $LA = (2 ft + 2 ft + 3 ft) \cdot 4 \frac{1}{2} ft$ $b = 3 f t^2$ $SA = 31 \frac{1}{2} f t^2 + 6 f t^2$ $LA = 7 ft \cdot 4\frac{1}{2} ft$ $SA = 37\frac{1}{2} ft^2$ $LA = 28\,ft^2 + 3\frac{1}{2}\,ft^2$ $LA = 31\frac{1}{2} ft^2$ The surface area of the right rectangular prism is $37\frac{1}{2}$ ft². The surface area of a right rectangular prism is $68\frac{2}{3}$ in². The dimensions of its base are 3 in and 7 in. Use the 6. formula SA = LA + 2B and LA = Ph to find the unknown height h of the prism. SA = LA + 2B $SA = P \cdot h + 2B$ $68\frac{2}{3}in^2 = 20in \cdot (h) + 2(21in^2)$ $68\frac{2}{2}in^2 = 20in \cdot (h) + 42in^2$ $68\frac{2}{2}in^2 - 42in^2 = 20in \cdot (h) + 42in^2 - 42in^2$ $26\frac{2}{3}in^2 = 20in \cdot (h) + 0in^2$ $26\frac{2}{3}in^2 \cdot \frac{1}{20in} = 20in \cdot \frac{1}{20in} \cdot (h)$ $\frac{80}{3}in^2\cdot\frac{1}{20in}=1\cdot h$ $\frac{4}{3}$ in = h $h = \frac{4}{3}$ in $= 1\frac{1}{3}$ in The height of the prism is $1\frac{1}{3}$ in.









