Lesson 7: Understanding Equations

Student Outcomes

- Students understand that an equation is a statement of equality between two expressions.
- Students build an algebraic expression using the context of a word problem and use that expression to write an equation that can be used to solve the word problem.

Related Topics: More Lesson Plans for Grade 7 Common Core Math

Lesson Notes

Students are asked to substitute a number for the variable to check whether it is a solution to the equation.

This lesson focuses on students building an equation that can be used to solve a word problem. The variable (letter) in an equation is a placeholder for a number. The equations might be true for some numbers and false for others. A solution to an equation is a number that makes the equation true when students are asked to substitute a number for the variable to check whether it is a solution to the equation. The emphasis of this lesson is for students to build an algebraic expression and set it equal to a number to form an equation that can be used to solve a word problem. As part of the activity, students are asked to check whether a number (or set of numbers) is a solution to the equation. Solving an equation algebraically is left for future lessons.

The definitions presented below form the foundation of the next few lessons in this topic. Please review these carefully to help you understand the structure of the lessons.

Equation: An *equation* is a statement of equality between two expressions.

If A and B are two expressions in the variable x, then $A = B$ is an equation in the variable x.

Students sometimes have trouble keeping track of what is an expression and what is an equation. An expression never includes an equal sign (=) and can be thought of as part of a sentence. The expression $3 + 4$ read aloud is, "Three plus four," which is only a phrase in a possible sentence. Equations, on the other hand, always have an equal sign, which is a symbol for the verb "is." The equation $3 + 4 = 7$ read aloud is, "Three plus four is seven," which expresses a complete thought, i.e., a sentence.

Number sentences—equations with numbers only—are special among all equations.

Number Sentence: A *number sentence* is a statement of equality (or inequality) between two numerical expressions.

A number sentence is by far the most concrete version of an equation. It also has the very important property that it is always true or always false, and it is this property that distinguishes it from a generic equation. Examples include $3 + 4 = 7$ (true) and $3 + 3 = 7$ (false). This important property guarantees the ability to check whether or not a number is a solution to an equation with a variable: just substitute a number into the variable. The resulting *number sentence* is either true or it is false. If the number sentence is true, the number is a solution to the equation. For that reason, number sentences are the first and most important type of equation that students need to understand.[1]

[1] Note that "sentence" is a legitimate mathematical term, not just a student-friendly version of "equation" meant for 1st or 2nd grade. To see what the term ultimately becomes, take a look at: mathworld.wolfram.com/Sentence.html.

Of course, we are mostly interested in numbers that make equations into true number sentences, and we have a special name for such numbers: a *solution.*

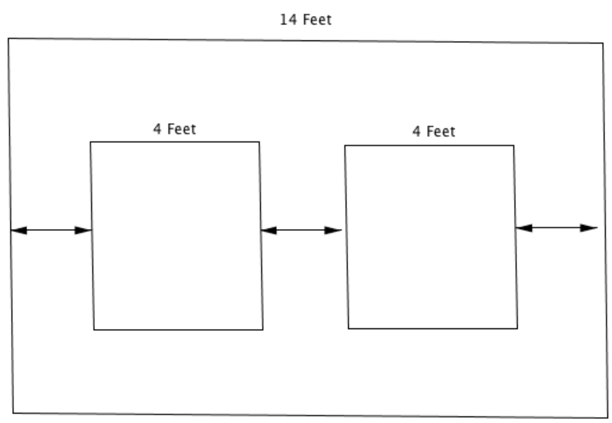
Solution: A *solution* to an equation with one variable is a number that, when substituted for all instances of the variable in both expressions, makes the equation a true number sentence.

Classwork

Opening Exercise (10 minutes)

Opening Exercise

Your brother is going to college, so you no longer have to share a bedroom. You decide to redecorate a wall by hanging two new posters on the wall. The wall is 14 feet wide, and each poster is four feet wide. You want to place the posters on the wall so that the distance from the edge of each poster to the nearest edge of the wall is the same as the distance between the posters, as shown in the diagram below. Determine that distance.



$14 - 4 - 4 = 6$

$6 \div 3 = 2$

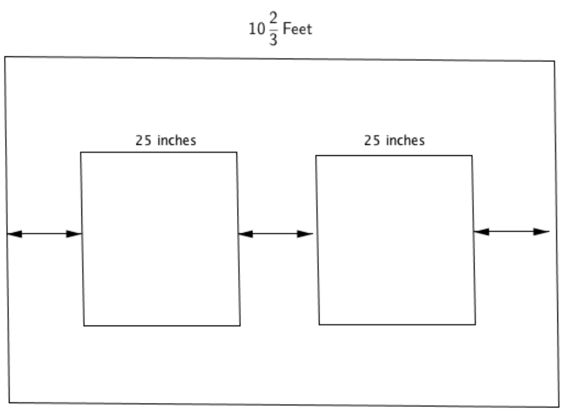
The distance is 2 feet.

Discussion

- Convey to students that the goal of this lesson is to learn how to build expressions and then write equations from those expressions. First, using the fact that the distance between the wall and poster is two feet, write an expression (in terms of the distances between and the width of the posters) for the total length of the wall.
  - $2 + 4 + 2 + 4 + 2$ or $3(2) + 4 + 4$ or $3(2) + 8$
- The numerical expressions you just wrote are based upon knowing what the answer is already. Suppose we wanted to solve the problem using algebra and did not know the answer is two feet. Let the distance between a picture and the nearest edge of the wall be x feet. Write an expression for the total length of such a wall in terms of x.
  - $x + 4 + x + 4 + x$ or $3x + 4 + 4$ or $3x + 8$

- Setting this expression equal to the total length of the wall described in the problem, 14 feet, gives an equation. Write that equation.
  - $x + 4 + x + 4 + x = 14$
  - $3x + 4 + 4 = 14$
  - $3x + 8 = 14$
- Using your answer from the Opening Exercise, check to see if your answer makes the equation true or false. Is the calculated distance consistent with the diagram that was drawn?
  - *If students calculated an answer of two feet, then the equation would be true; all other values for x will make the equation false.*
- We say that 2 is a *solution to* the equation $3x + 8 = 14$ because when it is substituted into the equation for x, it makes the equation a true number sentence: $3(2) + 8 = 14$.

Your parents are redecorating the dining room and want to place two rectangular wall sconce lights that are 25 inches wide along a 10 foot, 8 inch wall, so that the distance between the lights and the distances from each light to the nearest edge of the wall are all the same. Design the wall and determine the distance.



Scaffolding:
Review that 12 inches = 1 foot.

$$25\ inches = \frac{25}{12} = 2\frac{1}{12}\ feet$$
$$\left(10\frac{2}{3} - 2\frac{1}{12} - 2\frac{1}{12}\right) \div 3$$
$$\left(10\frac{8}{12} - 2\frac{1}{12} - 2\frac{1}{12}\right) \div 3$$
$$\left(6\frac{6}{12}\right) \div 3$$
$$6\frac{1}{2} \div 3 = \frac{13}{2} \div 3$$
$$\frac{13}{2} \times \frac{1}{3} = \frac{13}{6} = 2\frac{1}{6}\ feet$$

OR

$$10\frac{2}{3}\ feet = 10 \times 12 + \frac{2}{3} \times 12 = 120 + 8 = 128\ inches$$
$$(128 - 25 - 25) \div 3$$
$$78 \div 3$$
$$26\ inches$$

MP.7

Let the distance between a light and the nearest edge of a wall be x ft. Write an expression in terms of x for the total length of the wall, and then use the expression and the length of the wall given in the problem to write an equation that can be used to find that distance.

$3x + 2\frac{1}{12} + 2\frac{1}{12}$

$3x + 2\frac{1}{12} + 2\frac{1}{12} = 10\frac{2}{3}$

Now write an equation where y stands for the number of *inches*: Let the distance between a light and the nearest edge of a wall be y in. Write an expression in terms of y for the total length of the wall, and then use the expression and the length of the wall (in inches) given in the problem to write an equation that can be used to find that distance (in inches).

$2\frac{1}{12}$ *feet* $=$ 25 *inches; therefore, the expression is* $3y + 25 + 25$.

$10\frac{2}{3}$ *feet* $=$ 128 *inches; therefore, the equation is* $3y + 25 + 25 = 128$.

What value(s) of y makes the second equation true: 24, 25, or 26?

$y = 24$	$y = 25$	$y = 26$
$3y + 25 + 25 = 128$	$3y + 25 + 25 = 128$	$3y + 25 + 25 = 128$
$3(24) + 25 + 25 = 128$	$3(25) + 25 + 25 = 128$	$3(26) + 25 + 25 = 128$
$72 + 25 + 25 = 128$	$75 + 25 + 25 = 128$	$78 + 25 + 25 = 128$
$122 = 128$	$125 = 128$	$128 = 128$
False	*False*	*True*

Since substituting 26 *for* y *results in a true equation, the distance between the light and the nearest edge of the wall should be* 26 in.

Discussion (5 minutes)

- How did the change in the dimensions on the second problem change how you approached the problem?
  - *The fractional width of* $10\frac{2}{3}$ *feet makes the arithmetic more difficult, and the width of the posters expressed in a different unit than the width of the room makes the problem more difficult.*
- Since this problem is more difficult than the first, what additional steps are required to solve the problem?
  - *The widths must be in the same units. Therefore, you must either convert* $10\frac{2}{3}$ *feet to inches or convert* 25 *inches to feet.*
- Describe the process of converting the units.
  - *To convert* 25 *inches to feet, divide the* 25 *inches by* 12*, and write the quotient as a mixed number. To convert* $10\frac{2}{3}$ *feet to inches, multiply the whole number* 10 *by* 12*, and then multiply the fractional part,* $\frac{2}{3}$*, by* 12*; add the parts together to get the total number of inches.*
- After looking at both of the arithmetic solutions, which one seemed most efficient and why?
  - *Converting the width of the room from feet to inches made the overall problem shorter and easier because after converting the width, all of the dimensions ended up being whole numbers and not fractions.*

- Does it matter which equation you use when determining which given values make the equation true? Explain how you know.
  - *Yes, since the values were given in inches, the equation* $3y + 25 + 25 = 128$ *can be used because each term of the equation is in the same unit of measure.*
- If one uses the other equation, what must be done to obtain the solution?
  - *If the other equation were used, then the given values of* 24, 25, *and* 26 *inches need to be converted to* 2, $2\frac{1}{12}$, *and* $2\frac{1}{6}$ *feet, respectively.*

Example 1 (10 minutes)

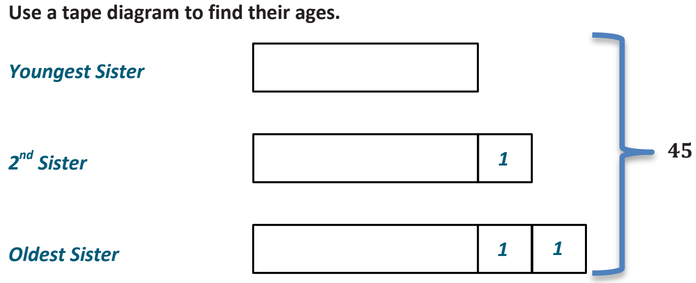
The example is a consecutive integer word problem. A tape diagram is used to model an arithmetic solution in part (a). Replacing the first bar (the youngest sister's age) in the tape diagram with x years provides an opportunity for students to visualize the meaning of the equation created in part (b).

Example 1

The ages of three sisters are consecutive integers. The sum of their ages is 45. Find their ages.

MP.4

a. **Use a tape diagram to find their ages.**



$45 - 3 = 42.$

$42 \div 3 = 14.$

Youngest Sister: 14 *years old*

2nd Sister: 15 *years old*

Oldest Sister: 16 *years old*

b. **If the youngest sister is x years old, describe the ages of the other two sisters in terms of x, write an expression for the sum of their ages in terms of x, and use that expression to write an equation that can be used to find their ages.**

Youngest Sister: x *years old*

2nd Sister: $(x + 1)$ *years old*

Oldest Sister: $(x + 2)$ *years old*

Sum of their ages: $x + (x + 1) + (x + 2)$

Equation: $x + (x + 1) + (x + 2) = 45$

c. **Determine if your answer from part (a) is a solution to the equation you wrote in part (b).**

$x + (x + 1) + (x + 2) = 45$
$14 + (14 + 1) + (14 + 2) = 45$
$45 = 45$

True

Scaffolding:

Review what is meant by consecutive integers–positive and negative whole numbers that increase or decrease by 1 unit. For example: $-2, -1, 0$.

- Let x be an integer; write an algebraic expression that represents one more than that integer.
  - $x + 1$
- Write an algebraic expression that represents two more than that integer.
  - $x + 2$

Discuss how the unknown unit in a tape diagram represents the unknown integer, represented by x. Consecutive integers begin with the unknown unit; then, every consecutive integer thereafter increases by 1 unit.

Exercise (8 minutes)

Instruct students to complete the following exercise individually and discuss the solution as a class.

Exercise

1. **Sophia pays a $\$19.99$ membership fee for an online music store.**

   a. **If she also buys two songs from a new album at a price of $\$0.99$ each, what is the total cost?**

   $\$21.97$

   b. **If Sophia purchases n songs for $\$0.99$ each, write an expression for the total cost.**

   $0.99n + 19.99$

   c. **Sophia's friend has saved $\$118$ but isn't sure how many songs she can afford if she buys the membership and some songs. Use the expression in part (b) to write an equation that can be used to determine how many songs Sophia's friend can buy.**

   $0.99n + 19.99 = 118$

   d. **Using the equation written in part (c), can Sophia's friend buy 101, 100, or 99 songs?**

$n = 99$	$n = 100$	$n = 101$
$0.99n + 19.99 = 118$	$0.99n + 19.99 = 118$	$0.99n + 19.99 = 118$
$0.99(99) + 19.99 = 118$	$0.99(100) + 19.99 = 118$	$0.99(101) + 19.99 = 118$
$98.01 + 19.99 = 118$	$99 + 19.99 = 118$	$99.99 + 19.99 = 118$
$118 = 118$	$118.99 = 118$	$119.98 = 118$
True	*False*	*False*

Closing (3 minutes)

- Describe the process you used today to create an equation: What did you build first? What did you set it equal to?
- Describe how to determine if a number is a solution to an equation.

Relevant Vocabulary

Variable (description): A *variable* is a symbol (such as a letter) that represents a number, i.e., it is a placeholder for a number.

Equation: An *equation* is a statement of equality between two expressions.

Number Sentence: A *number sentence* is a statement of equality between two numerical expressions.

Solution: A *solution* to an equation with one variable is a number that, when substituted for the variable in both expressions, makes the equation a true number sentence.

Lesson Summary

In many word problems, an equation is often formed by setting an expression equal to a number. To build the expression, it is often helpful to consider a few numerical calculations with just numbers first. For example, if a pound of apples costs $\$2$, then three pounds cost $\$6$ (2×3), four pounds cost $\$8$ (2×4), and n pounds cost $2n$ dollars. If we had $\$15$ to spend on apples and wanted to know how many pounds we could buy, we can use the expression $2n$ to write an equation, $2n = 15$, which can then be used to find the answer: $7\frac{1}{2}$ pounds.

To determine if a number is a solution to an equation, substitute the number into the equation for the variable (letter) and check to see if the resulting number sentence is true. If it is true, then the number is a solution to the equation. For example, $7\frac{1}{2}$ is a solution to $2n = 15$ because $2\left(7\frac{1}{2}\right) = 15$.

Exit Ticket (7 minutes)

Name ____ Date____________________

Lesson 7: Understanding Equations

Exit Ticket

1. Check whether the given value of x is a solution to the equation. Justify your answer.

   a. $\frac{1}{3}(x+4)=20$ $\qquad$ $x=48$

   b. $3x-1=5x+10$ $\qquad$ $x=-5\frac{1}{2}$

2. The total cost of four pens and seven mechanical pencils is $13.25. The cost of each pencil is 75 cents.

   a. Using an arithmetic approach, find the cost of a pen.

   b. Let the cost of a pen be p dollars. Write an expression for the total cost of four pens and seven mechanical pencils in terms of p.

c. Write an equation that could be used to find the cost of a pen.

d. Determine a value for p for which the equation you wrote in part (b) is true.

e. Determine a value for p for which the equation you wrote in part (b) is false.

Exit Ticket Sample Solutions

1. Check whether the given value of x is a solution to the equation. Justify your answer.

   a. $\frac{1}{3}(x+4)=20$ $\quad x=48$

   $\frac{1}{3}(48+4)=20$

   $\frac{1}{3}(52)=20$

   $17\frac{1}{3}=20$

   False, 48 is NOT a solution to $\frac{1}{3}(x+4)=20$.

   b. $3x-1=5x+10$ $\quad x=-5\frac{1}{2}$

   $3\left(-5\frac{1}{2}\right)-1=5\left(-5\frac{1}{2}\right)+10$

   $-\frac{33}{2}-1=-\frac{55}{2}+10$

   $-\frac{35}{2}=-\frac{35}{2}$

   True, $-5\frac{1}{2}$ *is a solution to* $3x-1=5x+10$.

2. The total cost of four pens and seven mechanical pencils is $\$13.25$. The cost of each pencil is 75 cents.

   a. Using an arithmetic approach, find the cost of a pen.

   $(13.25-7(0.75))\div 4$

   $(13.25-5.25))\div 4$

   $8\div 4$

   2

   b. Let the cost of a pen be p dollars. Write an expression for the total cost of four pens and seven mechanical pencils in terms of p.

   $4p+7(0.75)$ *or* $4p+5.25$

   c. Write an equation that could be used to find the cost of a pen.

   $4p+7(0.75)=13.25$ *or* $4p+5.25=13.25$

   d. Determine a value for p for which the equation you wrote in part (b) is true.

   $4p+5.25=13.25$

   $4(2)+5.25=13.25$

   $8+5.25=13.25$

   $13.25=13.25$

   True, when $p=2$*, the equation is true.*

   e. Determine a value for p for which the equation you wrote in part (b) is false.

   Any value other than 2 *will make the equation false.*

Problem Set Sample Solutions

1. Check whether the given value is a solution to the equation.

   a. $4n - 3 = -2n + 9$ $\qquad n = 2$

   True

   b. $9m - 19 = 3m + 1$ $\qquad m = \frac{10}{3}$

   True

   c. $3(y + 8) = 2y - 6$ $\qquad y = 30$

   False

2. Tell whether each number is a solution to the problem modeled by the following equation.

   Mystery Number: Five more than -8 times a number is 29. What is the number?

   Let the mystery number be represented by n.

   The equation is: $5 + (-8)n = 29$.

   a. Is 3 a solution to the equation? Why or why not?

   No, because $5 - 24 \neq 29$.

   b. Is -4 a solution to the equation? Why or why not?

   No, because $5 + 32 \neq 29$.

   c. Is -3 a solution to the equation? Why or why not?

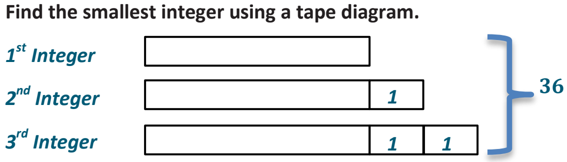
   Yes, because $5 + 24 = 29$.

   d. What is the mystery number?

   -3 *because* 5 *more than* -8 *times* -3 *is* 29.

3. The sum of three consecutive integers is 36.

   a. Find the smallest integer using a tape diagram.



   $36 - 3 = 33$

   $33 \div 3 = 11$

   The smallest integer is 11.

b. Let n represent the smallest integer. Write an equation that can be used to find the smallest integer.

Smallest integer: n

2nd integer: $(n + 1)$

3rd integer: $(n + 2)$

Sum of the three consecutive integers: $n + (n + 1) + (n + 2)$

Equation: $n + (n + 1) + (n + 2) = 36$

c. Determine if each value of n below is a solution to the equation in part (b).

$n = 12.5$	*No, it is not an integer and it does not make a true equation.*
$n = 12$	*No, it does not make a true equation.*
$n = 11$	*Yes, it makes a true equation.*

4. Andrew is trying to create a number puzzle for his younger sister to solve. He challenges his sister to find the mystery number. "When 4 is subtracted from half of a number the result is 5." The equation to represent the mystery number is $\frac{1}{2}m - 4 = 5$. Andrew's sister tries to guess the mystery number.

a. Her first guess is 30. Is she correct? Why or why not?

No, it does not make a true equation.

b. Her second guess is 2. Is she correct? Why or why not?

No, it does not make a true equation.

c. Her final guess is $4\frac{1}{2}$. Is she correct? Why or why not?

No, it does not make a true equation.