Lesson 5: Using the Identity and Inverse to Write Equivalent Expressions

Student Outcomes

- Students recognize the identity properties of 0 and 1 and the existence of inverses (opposites and reciprocals) to write equivalent expressions.

Related Topics: More Lesson Plans for Grade 7 Common Core Math

Classwork

Opening Exercise (5 minutes)

Students will work independently to rewrite numerical expressions recalling the definitions of opposites and reciprocals.

MP.8

Opening Exercise

1. **In the morning, Harrison checked the temperature outside to find that it was -12°F. Later in the afternoon, the temperature rose 12°F. Write an expression representing the temperature change. What was the afternoon temperature?**

   $-12 + 12$; the afternoon temperature was 0°F.

2. **Rewrite subtraction as adding the inverse for the following problems, and find the sum.**
   a. $2 - 2$

      $2 + (-2) = 0$

   b. $-4 - (-4)$

      $(-4) + 4 = 0$

   c. The difference of 5 and 5.

      $5 - 5 = 5 + (-5) = 0$

   d. $g - g$

      $g + (-g) = 0$

3. **What pattern can you deduce from Opening Exercises 1 and 2?**

   The sum of additive inverses equals zero.

4. **Add or subtract.**
   a. $16 + 0$

      16

b. $0 - 7$

$0 + (-7) = -7$

c. $-4 + 0$

-4

d. $0 + d$

d

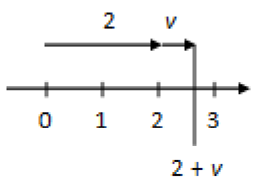
e. **What pattern do you notice in (a)–(d)?**

The sum of any quantity and zero is equal to the value of the quantity.

MP.8

5. **Your younger sibling runs up to you and excitedly exclaims, "I'm thinking of a number. If I add it to the number 2 ten times, that is, $2 +$ my number $+$ my number $+$ my number… and so on, then the answer is 2. What is my number?" You almost immediately answer, "zero," but are you sure? Can you find a different number (other than zero) that has the same property? If not, can you justify that your answer is the only correct answer?**

Answer: No, there is no other number. On a number line, 2 can be represented as a directed line segment that starts at 0, ends at 2, and has length 2. Adding any other (positive or negative) number v to 2 is equivalent to attaching another directed line segment with length $|v|$ to the end of the first line segment for 2:



If v is any number other than 0, then the directed line segment that represents v will have to have some length, so $2 + v$ will have to be a different number on the number line. Adding v again just takes the new sum further away from the point 2 on the number line.

Discussion (5 minutes)

Discuss the following questions and conclude the opening with definitions of *opposite*, *additive inverse,* and the *Identity Property of Zero*.

- In Problem 1, what is the pair of numbers called?
  - *Opposites or additive inverses.*
- What is the sum of a number and its opposite?
  - *It always equals to 0.*
- In Problem 5, what is so special about 0?
  - *Zero is the only number that when summed with another number, results in that number again. This property makes zero special among all the numbers, so special in fact, that mathematicians have a special name for zero, called the "additive identity"; they call that property the "Additive Identity Property of Zero."*

Example 1 (5 minutes)

As a class, write the sum and then write an equivalent expression by collecting like terms and removing parentheses when possible. State the reasoning for each step.

Example 1

Write the sum and then write an equivalent expression by collecting like terms and removing parentheses.

a. $2x$ and $-2x+3$

$2x+(-2x+3)$

$(2x+(-2x))+3$ — *Associative property, collect like-terms*

$0+3$ — *Additive inverse*

3 — *Additive identity property of zero*

b. $2x-7$ and the opposite of $2x$

$(2x-7)-2x$

$2x+(-7)+(-2x)$ — *Subtraction as adding the inverse*

$2x+(-2x)+(-7)$ — *Commutative property, associative property*

$0+(-7)$ — *Additive inverse*

-7 — *Additive identity property of zero*

c. The opposite of $(5x-1)$ and $5x$

$-(5x-1)+5x$

$-1(5x-1)+5x$ — *Taking the opposite is equivalent to multiplying by -1*

$-5x+1+5x$ — *Distributive property*

$(-5x+5x)+1$ — *Commutative property, any grouping property*

$0+1$ — *Additive inverse*

1 — *Additive identity property of zero*

Exercise 1 (10 minutes)

In pairs, students will take turns dictating how to write the sums while partners write what is being dictated. Students should discuss any discrepancies and explain their reasoning. Dialogue is encouraged.

Exercise 1

With a partner, take turns alternating roles as writer and speaker. The speaker verbalizes how to rewrite the sum and properties that justify each step as the writer writes what is being spoken without any input. At the end of each problem, discuss in pairs the resulting equivalent expressions.

Write the sum and then write an equivalent expression by collecting like terms and removing parentheses whenever possible.

a. -4 and $4b+4$

$-4+(4b+4)$

$(-4+4)+4b$ — *Any order, any grouping*

$0+4b$ — *Additive inverse*

$4b$ — *Additive identity property of zero*

b. $3x$ and $1-3x$

$3x+(1\ -3x)$

$3x+(1+(-3x))$ *Subtraction as adding the inverse*

$(3x+(-3x))+1$ *Any order, any grouping*

$0+1$ *Additive inverse*

1 *Additive identity property of zero*

c. The opposite of $4x$ and $-5\ +4x$

$-4x+(-5+4x)$

$(-4x+4x)+(-5)$ *Any order, any grouping*

$0+(-5)$ *Additive inverse*

-5 *Additive identity property of zero*

d. The opposite of $-10t$ and $t-10t$

$10t+(t-10t)$

$(10t+(-10t))+t$ *Any order, any grouping*

$0+t$ *Additive inverse*

t *Additive identity property of zero*

e. The opposite of $(-7-4v)$ and $-4v$

$-(-7-4v)+(-4v)$

$-1(-7-4v)+(-4v)$ *Taking the opposite is equivalent to multiplying by -1*

$7+4v+(-4v)$ *Distributive property*

$7+0$ *Any grouping, additive inverse*

7 *Additive identity property of zero*

Example 2 (5 minutes)

Students should complete the first five problems independently and discuss:

Example 2

- $\left(\frac{3}{4}\right)\times\left(\frac{4}{3}\right)$
- $4\times\frac{1}{4}$
- $\frac{1}{9}\times 9$
- $\left(-\frac{1}{3}\right)\times -3$
- $\left(-\frac{6}{5}\right)\times\left(-\frac{5}{6}\right)$

- What are these pairs of numbers called?
  - *Reciprocals.*
- What is another term for reciprocal?
  - *The multiplicative inverse.*
- What happens to the sign of the expression when converting it to its multiplicative inverse?
  - *There is no change to the sign. For example, the multiplicative inverse of* -2 *is* $\left(-\frac{1}{2}\right)$. *The negative sign remains the same.*
- What can you deduce from the pattern in the answers?
  - *The product of multiplicative inverses equals* 1.
- Earlier, we saw that 0 is a special number because it is the only number that when summed with another number, results in that number again. Can you explain why the number 1 is also special?
  - *Let students discuss in small groups and then as a class. Look for the answer, "One is the only number that when multiplied with another number, results in that number again." Then explain that this property makes* 1 *special among all the numbers, so special, in fact, that mathematicians have a special name for one, called the "multiplicative identity"; they call that property the "Multiplicative Identity Property of One."*
  - *As an extension, you can ask students if there are any other "special numbers" that they have learned. Yes:* -1 *has the property that multiplying a number by it is the same as taking the opposite of the number. Tell your students that they are going to learn later in this module about another special number called pi.*

As a class, write the product and then write an equivalent expression in standard form. State the properties for each step. After discussing questions, review the properties and definitions in the lesson summary emphasizing the Multiplicative Identity Property of 1 and the multiplicative inverse.

Write the product and then write the expression in standard form by removing parentheses and combining like terms. Justify each step.

a. **The multiplicative inverse of** $\frac{1}{5}$ **and** $\left(2x-\frac{1}{5}\right)$

$5\left(2x-\frac{1}{5}\right)$	
$10x-5\cdot\frac{1}{5}$	*Distributive property*
$10x-1$	*Multiplicative inverses*

b. **The multiplicative inverse of** 2 **and** $(2x+4)$

$\left(\frac{1}{2}\right)(2x+4)$	
$\left(\frac{1}{2}\right)(2x)+\left(\frac{1}{2}\right)(4)$	*Distributive property*
$1x+2$	*Multiplicative inverses, multiplication*
$x+2$	*Multiplicative identity property of one*

c. The multiplicative inverse of $\left(\frac{1}{3x+5}\right)$ and $\frac{1}{3}$

$(3x+5)\cdot\frac{1}{3}$

$3x\left(\frac{1}{3}\right)+5\left(\frac{1}{3}\right)$ *Distributive property*

$1x+\frac{5}{3}$ *Multiplicative inverse*

$x+\frac{5}{3}$ *Multiplicative identity property of one*

Exercise 2 (10 minutes)

As in Exercise 1, have students work in pairs taking turns being the speaker and writer rewriting the expressions.

Exercise 2

Write the product and then write the expression in standard form by removing parentheses and combining like terms. Justify each step.

a. The reciprocal of 3 and $-6y-3x$

$\left(\frac{1}{3}\right)(-6y+(-3x))$ *Rewrite subtraction as an addition problem*

$\left(\frac{1}{3}\right)(-6y)+\left(\frac{1}{3}\right)(-3x)$ *Distributive property*

$-2y-1x$ *Multiplicative inverse*

$-2y-x$ *Multiplicative identity property of one*

b. The multiplicative inverse of 4 and $4h-20$

$\left(\frac{1}{4}\right)(4h+(-20))$ *Rewrite subtraction as an addition problem*

$\left(\frac{1}{4}\right)(4h)+\left(\frac{1}{4}\right)(-20)$ *Distributive property*

$1h+(-5)$ *Multiplicative inverse*

$h-5$ *Multiplicative identity property of one*

c. The multiplicative inverse of $-\frac{1}{6}$ and $2-\frac{1}{6}j$

$(-6)\left(2+\left(-\frac{1}{6}j\right)\right)$ *Rewrite subtraction as an addition problem*

$(-6)(2)+(-6)\left(-\frac{1}{6}j\right)$ *Distributive property*

$-12+1k$ *Multiplicative inverse*

$-12+k$ *Multiplicative identity property of one*

Closing (3 minutes)

- What are the other terms for opposites and reciprocals, and what are the general rules of their sums and products?
  - *Additive inverse and multiplicative inverse; the sum of additive inverses equals* 0*; the product of multiplicative inverses equals* 1*.*
- What do the Additive Identity Property of Zero and the Multiplicative Identity Property of One state?
  - *The Additive Identity Property of Zero states that zero is the only number that when summed to another number, the result is again that number. The Multiplicative Identity Property of One states that one is the only number that when multiplied with another number, results in that number again.*

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 5: Using the Identity and Inverse to Write Equivalent Expressions

Exit Ticket

1. Find the sum of $5x + 20$ and the opposite of 20. Write an equivalent expression using the fewest number of terms. Justify each step.

2. For $5x + 20$ and the multiplicative inverse of 5, write the product and then write the expression in standard form, if possible. Justify each step.

Exit Ticket Sample Solutions

1. Find the sum of $5x + 20$ and the opposite of 20. Write an equivalent expression using the fewest number of terms. Justify each step.

$(5x + 20) + (-20)$

$5x + (20 + (-20))$ *Associative property of addition*

$5x + 0$ *Additive inverse*

$5x$ *Additive identity property of zero*

2. For $5x + 20$ and the multiplicative inverse of 5, write the product and then write the expression in standard form, if possible. Justify each step.

$(5x + 20)\left(\frac{1}{5}\right)$

$(5x)\left(\frac{1}{5}\right) + 20\left(\frac{1}{5}\right)$ *Distributive property*

$1x + 4$ *Multiplicative inverses, multiplication*

$x + 4$ *Multiplicative identity property of one*

Problem Set Sample Solutions

1. Fill in the missing parts of the worked out expressions.

a. The sum of $6c - 5$ and the opposite of $6c$

$(6c - 5) + (-6c)$

$(6c + (-5)) + (-6c)$ Rewrite subtraction as addition

$6c + (-6c) + (-5)$ *Regrouping/any order (or commutative property of addition)*

$0 + (-5)$ *Additive inverse*

-5 Additive identity property of zero

b. The product of $-2c + 14$ and the multiplicative inverse of -2

$(-2c + 14)\left(-\frac{1}{2}\right)$

$(-2c)\left(-\frac{1}{2}\right) + (14)\left(-\frac{1}{2}\right)$ *Distributive property*

$1c + (-7)$ Multiplicative inverse, multiplication

$1c - 7$ Adding the additive inverse is the same as subtraction

$c - 7$ *Multiplicative identity property of one*

2. Write the sum and then rewrite the expression in standard form by removing parentheses and collecting like terms.

a. 6 and $p - 6$

$6 + (p - 6)$

$6 + (-6) + p$

$0 + p$

p

b. $10w + 3$ and -3

$(10x + 3) + (-3)$

$10w + \left(3 + (-3)\right)$

$10w + 0$

$10w$

c. $-x - 11$ and the opposite of -11

$\left(-x + (-11)\right) + 11$

$-x + \left((-11) + (11)\right)$

$-x + 0$

$-x$

d. The opposite of $4x$ and $3 + 4x$

$(-4x) + (3 + 4x)$

$\left((-4x) + 4x\right) + 3$

$0 + 3$

3

e. $2g$ and the opposite of $(1 - 2g)$

$2g + \left(-(1 - 2g)\right)$

$2g + (-1) + 2g$

$2g + 2g + (-1)$

$4g + (-1)$

$4g - 1$

3. Write the product and then rewrite the expression in standard form by removing parentheses and collecting like terms.

a. $7h - 1$ and the multiplicative inverse of 7

$\left(7h + (-1)\right)\left(\frac{1}{7}\right)$

$\left(\frac{1}{7}\right)(7h) + \left(\frac{1}{7}\right)(-1)$

$h - \frac{1}{7}$

b. The multiplicative inverse of -5 and $10v - 5$

$\left(-\frac{1}{5}\right)(10v - 5)$

$\left(-\frac{1}{5}\right)(10v) + \left(-\frac{1}{5}\right)(-5)$

$-2v + 1$

c. $9 - b$ and the multiplicative inverse of 9

$(9 + (-b))\left(\frac{1}{9}\right)$

$\left(\frac{1}{9}\right)(9) + \left(\frac{1}{9}\right)(-b)$

$1 - \frac{1}{9}b$

d. The multiplicative inverse of $\frac{1}{4}$ and $5t - \frac{1}{4}$

$4\left(5t - \frac{1}{4}\right)$

$4(5t) + 4\left(-\frac{1}{4}\right)$

$20t - 1$

e. The multiplicative inverse of $-\frac{1}{10x}$ and $\frac{1}{10x} - \frac{1}{10}$

$(-10x)\left(\frac{1}{10x} - \frac{1}{10}\right)$

$(-10x)\left(\frac{1}{10x}\right) + (-10x)\left(-\frac{1}{10}\right)$

$-1 + x$

4. Write the expressions in standard form.

a. $\frac{1}{4}(4x + 8)$

$\frac{1}{4}(4x) + \frac{1}{4}(8)$

$x + 2$

b. $\frac{1}{6}(r - 6)$

$\frac{1}{6}(r) + \frac{1}{6}(-6)$

$\frac{1}{6}r - 1$

c. $\frac{4}{5}(x + 1)$

$\frac{4}{5}(x) + \frac{4}{5}(1)$

$\frac{4}{5}x + \frac{4}{5}$

d. $\frac{1}{8}(2x+4)$

$\frac{1}{8}(2x)+\frac{1}{8}(4)$

$\frac{1}{4}x+\frac{1}{2}$

e. $\frac{3}{4}(5x-1)$

$\frac{3}{4}(5x)+\frac{3}{4}(-1)$

$\frac{15}{4}x-\frac{3}{4}$

f. $\frac{1}{5}(10x-5)-3$

$\frac{1}{5}(10x)+\frac{1}{5}(-5)+(-3)$

$2x+(-1)+(-3)$

$2x-4$