Lesson 1: Generating Equivalent Expressions

Student Outcomes

- Students generate equivalent expressions using the fact that addition and multiplication can be done in *any order* (commutative property) and *any grouping* (associative property).
- Students recognize how *any order, any grouping* can be applied in a subtraction problem by using additive inverse relationships (adding the opposite) to form a sum and likewise with division problems by using the multiplicative inverse relationships (multiplying by the reciprocal) to form a product.
- Students recognize that *any order* does not apply to expressions mixing addition and multiplication, leading to the need to follow the order of operations.

Related Topics: More Lesson Plans for Grade 7 Common Core Math

Lesson Notes

The *any order any grouping* property introduced in this lesson combines the commutative and associative properties of addition, and it combines the commutative and associative properties of multiplication. The commutative and associative properties are regularly used in sequence to rearrange terms in an expression without necessarily making changes to the terms themselves. Therefore, students utilize the any order, any grouping property as a tool of efficiency for manipulating expressions. The any order, any grouping property is referenced in the Progressions for the Common Core State Standards in Mathematics: Grades 6–8, Expressions and Equations.

The definitions presented below related to variables and expressions form the foundation of the next few lessons in this topic. Please review these carefully in order to understand the structure of Topic A lessons.

Variable: A variable is a symbol (such as a letter) that represents a number, i.e., it is a placeholder for a number.

A variable is actually quite a simple idea: it is a placeholder—a blank—in an expression or an equation where a number can be inserted. A variable holds a place for *a single number* throughout all calculations done with the variable—it does not vary. It is the *user of the variable* who has ultimate power to change or vary what number is inserted, *as he/she desires*. The power to "vary" rests in the will of the student, not in the variable itself.

Numerical Expression (in middle school): A numerical expression is a number, or it is any combination of sums, differences, products, or divisions of numbers that evaluates to a number.

Statements such as "$3 +$" or "$3 \div 0$" are not numerical expressions because neither represents a point on the number line.

Value of a Numerical Expression: The value of a numerical expression is the number found by evaluating the expression.

For example, $\frac{1}{3} \cdot (2 + 4) - 7$ is a numerical expression, and its value is -5. Note to teachers: Please do not stress words over meaning here; it is okay to use "number computed," "computation," "calculation," etc. to refer to the value as well.

Expression (in middle school): An *expression* is a numerical expression, or it is the result of replacing some (or all) of the numbers in a numerical expression with variables.

There are two ways to build expressions:

- We can start out with a numerical expression, such as $\frac{1}{3} \cdot (2 + 4) - 7$, and replace some of the numbers with letters to get $\frac{1}{3} \cdot (x + y) - z$.
- We can build such expressions from scratch, as in $x + x(y - z)$, and note that if numbers were placed in the expression for the variables x, y, and z, the result would be a numerical expression.

The key is to strongly link expressions back to computations with numbers through building and evaluating them. Building an expression often occurs in the context of a word problem by thinking about examples of numerical expressions first, and then replacing some of the numbers with letters in a numerical expression. The act of evaluating an expression means to replace each of the variables with specific numbers to get a numerical expression, and then finding the value of that numerical expression.

The description of expression above is meant to work nicely with how students in 6^{th} and 7^{th} grades learn to manipulate expressions. In these grades, students spend a lot of time building and evaluating expressions for specific numbers substituted into the variables. Building and evaluating helps students see that expressions are really just a slight abstraction of arithmetic in elementary school.

Equivalent Expressions (in middle school): Two expressions are *equivalent* if both expressions evaluate to the same number for every substitution of numbers into all the letters in both expressions. This description becomes clearer through lots of examples and linking to the associative, commutative, and distributive properties.

An Expression in Expanded Form (in middle school): An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in *expanded form*.

An Expression in Standard Form (in middle school): An expression that is in expanded form where all like-terms have been collected is said to be in *standard form*.

IMPORTANT: An expression in *standard form* is the equivalent of what is traditionally referred to as a "simplified" expression. This curriculum does not utilize the term "simplify" when writing equivalent expressions, but rather asks students to "put an expression in standard form" or "expand the expression and combine like terms." However, students must know that the term "simplify" will be seen outside of this curriculum and that the term is directing them to write an expression in standard form.

Lesson materials preparation: Prepare a classroom set of manila envelopes (non-translucent). Cut and place four triangles and two quadrilaterals in each envelope (provided at the end of this lesson). These envelopes are used in the Opening Exercise of this lesson.

Classwork

Opening Exercise (15 minutes)

This exercise requires students to represent unknown quantities with variable symbols and reason mathematically with those symbols to represent another unknown value.

As students enter the classroom, provide each one with an envelope containing two quadrilaterals and four triangles; instruct students not to open their envelopes. Divide students into teams of two to complete parts (a) and (b).

MP.2

Opening Exercise

Each envelope contains a number of triangles and a number of quadrilaterals. For this exercise, let t represent the number of triangles, and let q represent the number of quadrilaterals.

a. Write an expression, using t and q, that represents the total number of sides in your envelope. Explain what the terms in your expression represent.

$3t + 4q$. Triangles have 3 sides, so there will be 3 sides for each triangle in the envelope. This is represented by $3t$. Quadrilaterals have 4 sides, so there will be 4 sides for each quadrilateral in the envelope. This is represented by $4q$. The total number of sides will be the number of triangle sides and the number of quadrilateral sides together.

b. You and your partner have the same number of triangles and quadrilaterals in your envelopes. Write an expression that represents the total number of sides that you and your partner have. If possible, write more than one expression to represent this total.

$3t + 4q + 3t + 4q$; $2(3t + 4q)$; $6t + 8q$

Scaffolding:

To help students understand the given task, discuss a numerical expression, such as

$$2 \times 3 + 6 \times 4$$

as an example where there are two triangles and six quadrilaterals.

Discuss the variations of the expression in part (b) and whether those variations are equivalent. This discussion helps students understand what it means to combine like terms; some students have added their number of triangles together and number of quadrilaterals together, while others simply doubled their own number of triangles and quadrilaterals since the envelopes contain the same number. This discussion further shows how these different forms of the same expression relate to each other. Students then complete part (c).

MP.8

c. Each envelope in the class contains the same number of triangles and quadrilaterals. Write an expression that represents the total number of sides in the room.

Answer depends on the seat size of the classroom. For example, if there are 12 students in the class, the expression would be $12(3t + 4q)$, or an equivalent expression.

Next, discuss any variations (or possible variations) of the expression in part (c), and discuss whether those variations are equivalent. Are there as many variations in part (c), or did students use multiplication to consolidate the terms in their expressions? If the latter occurred, discuss the students' reasoning.

Choose one student to open his/her envelope and count the numbers of triangles and quadrilaterals. Record the values of t and q as reported by that student for all students to see. Next, students complete parts (d), (e), and (f).

d. **Use the given values of t and q, and your expression from part (a), to determine the number of sides that should be found in your envelope.**

$3t + 4q$
$3(4) + 4(2)$
$12 + 8$
20

There should be 20 sides contained in my envelope.

e. **Use the same values for t and q, and your expression from part (b), to determine the number of sides that should be contained in your envelope and your partner's envelope combined.**

Variation #1	*Variation #2*	*Variation #3*
$2(3t + 4q)$	$3t + 4q + 3t + 4q$	$6t + 8q$
$2(3(4) + 4(2))$	$3(4) + 4(2) + 3(4) + 4(2)$	$6(4) + 8(2)$
$2(12 + 8)$	$12 + 8 + 12 + 8$	$24 + 16$
$2(20)$	$20 + 12 + 8$	40
40	40	

My partner and I have a combined total of 40 sides.

f. **Use the same values for t and q, and your expression from part (c), to determine the number of sides that should be contained in all of the envelopes combined.**

Answer will depend on the seat size of your classroom. Sample responses for a class size of 12:

Variation 1	*Variation 2*	*Variation 3*
$12(3t + 4q)$	$\overbrace{3t + 4q}^{1} + \overbrace{3t + 4q}^{2} + \cdots + \overbrace{3t + 4q}^{12}$	$36t + 48q$
$12(3(4) + 4(2))$	$\overbrace{3(4) + 4(2)}^{1} + \overbrace{3(4) + 4(2)}^{2} + \cdots + \overbrace{3(4) + 4(2)}^{12}$	$36(4) + 48(2)$
$12(12 + 8)$	$\overbrace{3(4) + 4(2)}^{1} + \overbrace{3(4) + 4(2)}^{2} + \cdots + \overbrace{3(4) + 4(2)}^{12}$	$144 + 96$
$12(20)$	$\overbrace{12 + 8}^{1} + \overbrace{12 + 8}^{2} + \cdots + \overbrace{12 + 8}^{12}$	240
240	$\overbrace{20}^{1} + \overbrace{20}^{2} + \cdots + \overbrace{20}^{12}$	240

For a class size of 12 students, there should be 240 sides in all of the envelopes combined.

Have all students open their envelopes and confirm that the number of triangles and quadrilaterals matches the values of t and q recorded after part (c). Then, have students count the number of sides contained on the triangles and quadrilaterals from their own envelope and confirm with their answer to part (d). Next, have partners count how many sides they have combined and confirm that number with their answer to part (e). Finally, total the number of sides reported by each student in the classroom and confirm this number with the answer to part (f).

g. **What do you notice about the various expressions in parts (e) and (f)?**

The expressions in part (e) are all equivalent because they evaluate to the same number: 40. The expressions in part (f) are all equivalent because they evaluate to the same number: 240. The expressions themselves all involve the expression $3t + 4q$ in different ways. In part (e), $3t + 3t$ is equivalent to $6t$, and $4q + 4q$ is equivalent to $8q$. There appear to be several relationships among the representations involving the commutative, associative, and distributive properties.

When finished, have students return their triangles and quadrilaterals to their envelopes for use by other classes.

Example 1 (10 minutes): Any Order, Any Grouping Property with Addition

This example examines how and why we combine numbers and other like terms in expressions. An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form*. A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include 324, $3x$, $5x + 3 - 40$, $x + 2x + 3x$, etc.

Each summand of an expression in expanded form is called a *term*, and the number found by multiplying just the numbers in a term together is called the *coefficient of the term.* After defining the word *term*, students can be shown what it means to "combine like terms" using the distributive property. Students saw in the Opening Exercise that terms sharing exactly the same letter could be combined by adding (or subtracting) the coefficients of the terms:

$$3t + 3t = \overbrace{(3+3)}^{coefficients} \cdot t = 6t, \qquad \text{and} \qquad 4q + 4q = \overbrace{(4+4)}^{coefficients} \cdot q = 8q.$$

An expression in expanded form with all its like terms collected is said to be in *standard form*.

Scaffolding:

Refer students to the triangles and quadrilaterals from the opening exercise to understand why terms containing the same variable symbol x can be added together into a single term.

Example 1: Any Order, Any Grouping Property with Addition

a. **Rewrite $5x + 3x$ and $5x - 3x$ by combining like terms.**

Write the original expressions and expand each term using addition. What are the new expressions equivalent to?

$$5x + 3x = \underbrace{\overbrace{x+x+x+x+x}^{5x} + \overbrace{x+x+x}^{3x}}_{8x} = 8x$$

$$5x - 3x = \overbrace{x+x+\underbrace{x+x+x}_{3x}}^{5x} = 2x$$

- Because both terms have the common factor of x, we can use the distributive property to create an equivalent expression.

$$5x + 3x \qquad\qquad 5x - 3x$$
$$(5+3)x = 8x \qquad (5-3)x = 2x$$

Scaffolding:

Note to the teacher: The distributive property was covered in Grade 6 (6.EE.3) and is reviewed here in preparation for further use in this module starting with Lesson 3.

Ask students to try to find an example (a value for x) where $5x + 3x \neq 8x$ or where $5x - 3x \neq 2x$. Encourage them to use a variety of positive and negative rational numbers. Their failure to find a counterexample will help students realize what equivalence means.

MP.7

In Example 1b, students see that the commutative and associative properties of addition are regularly used in consecutive steps to reorder and regroup like terms so that they can be combined. Because the use of these properties does not change the value of an expression or any of the terms within the expression, the commutative and associative properties of addition can be used simultaneously. The simultaneous use of these properties is referred to as the *any order, any grouping property*.

Scaffolding:

Teacher may also want to show the expression as:

$\underbrace{x+x+1}_{2x+1}+\underbrace{x+x+x+x+x}_{5x}$

in the same manner as part (a).

b. Find the sum of $2x+1$ and $5x$.

$(2x+1)+5x$	*Original expression*
$2x+(1+5x)$	*Associative property of addition*
$2x+(5x+1)$	*Commutative property of addition*
$(2x+5x)+1$	*Associative property of addition*
$(2+5)x+1$	*Combined like terms (the distributive property)*
$7x+1$	*Equivalent expression to the given problem*

With a firm understanding of the commutative and associative properties of addition, students further understand that these steps can be combined.

- Why did we use the associative and commutative properties of addition?
  - *We reordered the terms in the expression to group together like terms so that they could be combined.*
- Did the use of these properties change the value of the expression? How do you know?
  - *The properties did not change the value of the expression because each equivalent expression includes the same terms as the original expression, just in a different order and grouping.*
- If a sequence of terms is being added, the *any order, any grouping* property allows us to add those terms in any order by grouping them together in any way.
- How can we confirm that the expressions $(2x+1)+5x$ and $7x+1$ are equivalent expressions?
  - *When a number is substituted for the x in both expressions, they both should yield equal results.*

Teacher and student choose a number, such as 3, to substitute for the value of x and together check to see if both expressions evaluate to the same result.

Given Expression	*Equivalent Expression?*
$(2x+1)+5x$	$7x+1$
$(2\cdot 3+1)+5\cdot 3$	$7\cdot 3+1$
$(6+1)+15$	$21+1$
$(7)+15$	22
22	

The expressions both evaluate to 22; however, this is only one possible value of x. Challenge students to find a value for x for which the expressions do not yield the same number. Students find that the expressions evaluate to equal results no matter what value is chosen for x.

- What prevents us from using any order, any grouping in part (c) and what can we do about it?
  - *The second expression, $(5a-3)$, involves subtraction, which is not commutative or associative; however, subtracting a number x can be written as adding the opposite of that number. So, by changing subtraction to addition, we can use any order and any grouping.*

c. **Find the sum of $-3a+2$ and $5a-3$.**

$(-3a+2)+(5a-3)$	*Original expression*
$-3a+2+5a+(-3)$	*Add the opposite (additive inverse)*
$-3a+5a+2+(-3)$	*Any order, any grouping*
$2a+(-1)$	*Combined like terms (Stress to students that the expression is not yet simplified.)*
$2a-1$	*Adding the inverse is subtracting*

- What was the only difference between this problem and those involving all addition?
  - *We first had to rewrite subtraction as addition; then, this problem was just like the others.*

Example 2 (3 minutes): Any Order, Any Grouping with Multiplication

MP.7

Students relate a product to its expanded form and understand that the same result can be obtained using any order, any grouping since multiplication is also associative and commutative.

Example 2: Any Order, Any Grouping with Multiplication

Find the product of $2x$ and 3.

$2x\cdot 3=2x+2x+2x=6x$

$2\cdot(x\cdot 3)$	*Associative property of multiplication (any grouping)*
$2\cdot(3\cdot x)$	*Commutative property of multiplication (any order)*
$6x$	*Multiplication*

With a firm understanding of the commutative and associative properties of multiplication, students further understand that these steps can be combined.

- Why did we use the associative and commutative properties of multiplication?
  - *We reordered the factors to group together the numbers so that they could be multiplied.*
- Did the use of these properties change the value of the expression? How do you know?
  - *The properties did not change the value of the expression because each equivalent expression includes the same factors as the original expression, just in a different order or grouping.*
- If a product of factors is being multiplied, the *any order, any grouping* property allows us to multiply those factors in any order by grouping them together in any way.

Example 3 (9 minutes): Any Order, Any Grouping in Expressions with Addition and Multiplication

Students use any order, any grouping to rewrite products with a single coefficient first as terms only, then as terms within a sum, noticing that any order, any grouping cannot be used to mix multiplication with addition.

Example 3: Any Order, Any Grouping in Expressions with Addition and Multiplication

Use any order, any grouping to find equivalent expressions.

a. $3(2x)$

$(3\cdot 2)x$

$6x$

Ask students to try to find an example (a value for x) where $3(2x) \neq 6x$. Encourage them to use a variety of positive and negative rational numbers because in order for the expressions to be equivalent, the expressions must evaluate to equal numbers for *every* substitution of numbers into all the letters in both expressions. Again, the point is to help students recognize that they cannot find a value—that the two expressions are equivalent. Encourage students to follow the order of operations for the expression $3(2x)$: multiply by 2 first, then by 3.

b. $4y(5)$

$(4 \cdot 5)y$

$20y$

c. $4 \cdot 2 \cdot z$

$(4 \cdot 2)z$

$8z$

d. $3(2x) + 4y(5)$

$3(2x) + 4y(5) = \overbrace{2x + 2x + 2x}^{6x} + \overbrace{4y + 4y + 4y + 4y + 4y}^{20y}$

$(3 \cdot 2)x + (4 \cdot 5)y$

$6x + 20y$

e. $3(2x) + 4y(5) + 4 \cdot 2 \cdot z$

$3(2x) + 4y(5) + 4 \cdot 2 \cdot z = \overbrace{2x + 2x + 2x}^{6x} + \overbrace{4y + 4y + 4y + 4y + 4y}^{20y} + \overbrace{z + z + z + z + z + z + z + z}^{8z}$

$(3 \cdot 2)x + (4 \cdot 5)y + (4 \cdot 2)z$

$6x + 20y + 8z$

f. **Alexander says that $3x + 4y$ is equivalent to $(3)(4) + xy$ because of any order, any grouping. Is he correct? Why or why not?**

MP.3

Encourage students to substitute a variety of positive and negative rational numbers for x and y because in order for the expressions to be equivalent, the expressions must evaluate to equal numbers for *every* substitution of numbers into all the letters in both expressions.

Alexander is incorrect; the expressions are not equivalent because if we, for example, let $x = -2$ and let $y = -3$, then we get the following:

$3x + 4y$	$(3)(4) + xy$
$3(-2) + 4(-3)$	$12 + (-2)(-3)$
$-6 + (-12)$	$12 + 6$
-18	18

$-18 \neq 18$ ***so, the expressions cannot be equivalent.***

- What can be concluded as a result of part (f)?
  - *Any order, any grouping cannot be used to mix multiplication with addition. Numbers and letters that are factors within a given term must remain factors within that term.*

Closing (3 minutes)

- We found that we can use any order, any grouping of terms in a sum, or of factors in a product. Why?
  - *Addition and multiplication are both associative and commutative and these properties only change the order and grouping of terms in a sum or factors in a product without affecting the value of the expression.*
- Can we use any order, any grouping when subtracting expressions? Explain.
  - *We can use any order any grouping after rewriting subtraction as the sum of a number and the additive inverse of that number, so that the expression becomes a sum.*
- Why can't we use any order, any grouping in addition and multiplication at the same time?
  - *Multiplication must be completed before addition. If you mix the operations, you change the value of the expression.*

Relevant Vocabulary:

Variable (description): A *variable* is a symbol (such as a letter) that represents a number, i.e., it is a placeholder for a number.

Numerical Expression (description): A *numerical expression* is a number, or it is any combination of sums, differences, products, or divisions of numbers that evaluates to a number.

Value of a Numerical Expression: The *value of a numerical expression* is the number found by evaluating the expression.

Expression (description): An *expression* is a numerical expression, or it is the result of replacing some (or all) of the numbers in a numerical expression with variables.

Equivalent Expressions: Two expressions are *equivalent* if both expressions evaluate to the same number for every substitution of numbers into all the letters in both expressions.

An Expression in Expanded Form: An expression that is written as sums (and/or differences) of products whose factors are numbers, variables, or variables raised to whole number powers is said to be in *expanded form.* A single number, variable, or a single product of numbers and/or variables is also considered to be in expanded form. Examples of expressions in expanded form include: 324, $3x$, $5x + 3 - 40$, $x + 2x + 3x$, etc.

Term (description): Each summand of an expression in expanded form is called a *term.* For example, the expression $2x + 3x + 5$ consists of 3 terms: $2x$, $3x$, and 5.

Coefficient of the Term (description): The number found by multiplying just the numbers in a term together. For example, given the product $2 \cdot x \cdot 4$, its equivalent term is $8x$. The number 8 is called the coefficient of the term $8x$.

An Expression in Standard Form: An expression in expanded form with all its like terms collected is said to be in *standard form.* For example, $2x + 3x + 5$ is an expression written in expanded form; however, to be written in standard form, the like terms $2x$ and $3x$ must be combined. The equivalent expression $5x + 5$ is written in standard form.

Lesson Summary

Terms that contain exactly the same variable symbol can be combined by addition or subtraction because the variable represents the same number. Any order, any grouping can be used where terms are added (or subtracted) in order to group together like terms. Changing the orders of the terms in a sum does not affect the value of the expression for given values of the variable(s).

Exit Ticket (5 minutes)

Name ____ Date____________________

Lesson 1: Generating Equivalent Expressions

Exit Ticket

1. Write an equivalent expression to $2x + 3 + 5x + 6$ by combining like terms.

2. Find the sum of $(8a + 2b - 4)$ and $(3b - 5)$.

3. Write the expression in standard form: $4(2a) + 7(-4b) + (3 \cdot c \cdot 5)$.

Exit Ticket Sample Solutions

1. Write an equivalent expression to $2x + 3 + 5x + 6$ by combining like terms.

   $2x + 3 + 5x + 6$

   $2x + 5x + 3 + 6$

   $7x + 9$

2. Find the sum of $(8a + 2b - 4)$ and $(3b - 5)$.

   $(8a + 2b - 4) + (3b - 5)$

   $8a + 2b + (-4) + 3b + (-5)$

   $8a + 2b + 3b + (-4) + (-5)$

   $8a + (5b) + (-9)$

   $8a + 5b - 9$

3. Write the expression in standard form: $4(2a) + 7(-4b) + (3 \cdot c \cdot 5)$.

   $(4 \cdot 2)a + \big(7 \cdot (-4)\big)b + (3 \cdot 5)c$

   $8a + (-28)b + 15c$

   $8a - 28b + 15c$

Problem Set Sample Solutions

For problems 1–9, write equivalent expressions by combining like terms. Verify the equivalence of your expression and the given expression by evaluating each for the given values: $a = 2$, $b = 5$, and $c = -3$.

1. $3a + 5a$

   $8a$
   $8(2)$
   16

   $3(2) + 5(2)$
   $6 + 10$
   16

2. $8b - 4b$

   $4b$
   $4(5)$
   20

   $8(5) - 4(5)$
   $40 - 20$
   20

3. $5c + 4c + c$

   $10c$
   $10(-3)$
   -30

   $5(-3) + 4(-3) + (-3)$
   $-15 + (-12) + (-3)$
   $-27 + (-3)$
   -30

4. $3a + 6 + 5a$

   $8a + 6$
   $8(2) + 6$
   $16 + 6$
   22

   $3(2) + 6 + 5(2)$
   $6 + 6 + 10$
   $12 + 10$
   22

5. $8b + 8 - 4b$

   $4b + 8$
   $4(5) + 8$
   $20 + 8$
   28

   $8(5) + 8 - 4(5)$
   $40 + 8 - 20$
   $48 - 20$
   28

6. $5c - 4c + c$

   $2c$
   $2(-3)$
   -6

   $5(-3) - 4(-3) + (-3)$
   $-15 + \big(-4(-3)\big) + (-3)$
   $-15 + (12) + (-3)$
   $-3 + (-3)$
   -6

7. $3a+6+5a-2$

$8a\ +\ 4$
$8(2)+4$
$16+4$
20

$3(2)+6+5(2)-2$
$6+6+10+(-2)$
$12+10+(-2)$
$22+(-2)$
20

8. $8b+8-4b-3$

$4b+5$
$4(5)+5$
$20+5$
25

$8(5)+8-4(5)-3$
$40+8+\left(-4(5)\right)+(-3)$
$40+8+(-20)+(-3)$
$48+(-20)+(-3)$
$28+(-3)$
25

9. $5c-4c+c-3c$

$-1c$
$-1(-3)$
3

$5(-3)-4(-3)+(-3)-3(-3)$
$-15+\left(-4(-3)\right)+(-3)+\left(-3(-3)\right)$
$-15+(12)+(-3)+(9)$
$-3+(-3)+9$
$-6+9$
3

Use any order, any grouping to write equivalent expressions by combining like terms. Then verify the equivalence of your expression to the given expression by evaluating for the value(s) given in each problem.

Problem	*Your Expression*	*Given Expression*
10. $3(6a)$; for $a=3$ $18a$	$18a$ $18(3)$ 54	$3\left(6(3)\right)$ $3(18)$ 54
11. $5d(4)$; for $d=-2$ $20d$	$20d$ $20(-2)$ -40	$5(-2)(4)$ $-10(4)$ -40
12. $(5r)(-2)$; for $r=-3$ $-10r$	$-10r$ $-10(-3)$ 30	$\left(5(-3)\right)(-2)$ $(-15)(-2)$ 30
13. $3b(8)+(-2)(7c)$; for $b=2, c=3$ $24b-14c$	$24b-14c$ $24(2)-14(3)$ $48-42$ 6	$3(2)(8)+(-2)(7(3))$ $6(8)+(-2)(21)$ $48+(-42)$ 6
14. $-4(3s)+2(-t)$; for $s=\frac{1}{2},\ t=-3$ $-12s-2t$	$-12s-2t$ $-12\left(\frac{1}{2}\right)-2(-3)$ $-6+\left(-2(-3)\right)$ $-6+(6)$ 0	$-4\left(3\left(\frac{1}{2}\right)\right)+2\left(-(-3)\right)$ $-4\left(\frac{3}{2}\right)+2(3)$ $-2(3)+2(3)$ $-6+6$ 0
15. $9(4p)-2(3q)+p$; for $p=-1, q=4$ $37p-6q$	$37p-6q$ $37(-1)-6(4)$ $-37+\left(-6(4)\right)$ $-37+(-24)$ -61	$9\left(4(-1)\right)-2\left(3(4)\right)+(-1)$ $9(-4)+\left(-2(12)\right)+(-1)$ $-36+(-24)+(-1)$ $-60+(-1)$ -61
16. $7(4g)+3(5h)+2(-3g)$; $g=\frac{1}{2}, h=\frac{1}{3}$ $28g+15h+(-6g)$ $22g+15h$	$22g+15h$ $22\left(\frac{1}{2}\right)+15\left(\frac{1}{3}\right)$ $11+5$ 16	$7\left(4\left(\frac{1}{2}\right)\right)+3\left(5\left(\frac{1}{3}\right)\right)+2\left(-3\left(\frac{1}{2}\right)\right)$ $7(2)+3\left(\frac{5}{3}\right)+2\left(-\frac{3}{2}\right)$ $14+5+(-3)$ $19+(-3)$ 16

The problems below are follow-up questions to Example 1b from Classwork: Find the sum of $2x + 1$ and $5x$.

17. **Jack got the expression $7x + 1$, and then wrote his answer as $1 + 7x$. Is his answer an equivalent expression? How do you know?**

    Yes; Jack correctly applied any order (the commutative property), changing the order of addition.

18. **Jill also got the expression $7x + 1$, then wrote her answer as $1x + 7$. Is her expression an equivalent expression? How do you know?**

    No, "any order" (the commutative property) does not apply to mixing addition and multiplication; therefore, the $7x$ must remain intact as a term.

    $1(4) + 7 = 11$ *and* $7(4) + 1 = 29$*; the expressions do no evaluate to the same value for* $x = 4$*.*

Materials for Opening Exercise

Photo copy each page and cut out the triangles and quadrilaterals for use in the Opening Exercise.