## Lesson 21

Objective: Recognize and show that equivalent fractions refer to the same point on the number line.

Related Topics: More Lesson Plans for the Common Core Math

## Suggested Lesson Structure

| $\square$ | Fluency Practice |
| :--- | :--- |
| $\square$ Application Problem | (12 minutes) |
| Concept Development | $(30$ minutes) |
| $\square$ Student Debrief | $(10$ minutes) |
| Total Time | $(60$ minutes) |



## Fluency Practice ( 12 minutes)

- Whole Number Division 3.0A. 7 (8 minutes)
- 1 Whole Expressed as Unit Fractions 3.NF. 1 (4 minutes)


## Whole Number Division (8 minutes)

Timing Note: Steps 1 and 2 are timed for two minutes. Step 3 is timed for 1 minute of testing for each partner. Step 4 is timed for two minutes

Step 1: Students self-select a number and write a set of multiples up to that number's multiple of 10 vertically down the left hand side of the page (e.g. $6,12,18,24,30,36,42,48,54,60$ ).
Step 2: Divide the number by the multiple vertically down the page.
Step 3: Change papers and test a partner out of order, e.g. "What is 24 divided by 6?"
Step 4: $\quad$ Redo the process of steps 1 and 2 to see improvement.
Let the students know that the same activity will be done the next day so that they have a chance to practice and further improve, possibly advancing to the next number which might further challenge them.

## 1 Whole Expressed as Unit Fractions (4 minutes)

T : Show me a number bond which partitions a whole into 3 equal parts.
S: (Students show.)
T : What is the unit fraction?
S : 1 third.
Repeat with other examples.

## Application Problem (8 minutes)

Dorothea is training to run a 2 mile race. She marks off her starting point and the finish line. So that she can track her progress, she places a mark at 1 mile. She then places a mark halfway between her starting position and 1 mile, and another halfway between 1 mile and the finish line.

- Draw a number line to show what unit fraction she makes as she marks the points on her run.
- How far has she run when she gets to the third marker?


## Concept Development (30 minutes)


a) She makes fourths with the points on her ron.
b) She has run $\frac{3}{4}$ of the way between $O$ and 2 miles.

Materials: (S) Fraction strips ( $4 \frac{1}{4}$ inches by 1 inch), math journals, crayons, glue, tape, meter stick, number line on the floor

Each student has five fraction strips.
T: We're going to make copies of unit fractions with our fraction strips. Fold your first strip into halves.
S : (Students fold.)
T: Label each part with a unit fraction. Then use a crayon to shade in 1 half.
S: (Label and shade.)
T: Glue your fraction strip at the top of a new page in your math journal.
S: (Students glue.)
T: Fold another fraction strip to make copies of fourths. Label each part with a unit fraction. Then glue your fraction strip directly below the first one in your math journal. Make sure that the ends are lined up.
S: (Students fold, label, place, and glue fraction strips.)
T: Now shade the number of fourths that are equivalent to the shaded half. Whisper to your partner how many units you shaded.

## NOTES ON

MULTIPLE MEANS OF REPRESENTATION:

The vocabulary word 'equivalent' has the advantage of cognates in many languages. Build ELLs' understanding of 'equivalent fractions' through discussion, word webs, and questioning.
Ask:

- How are these equivalent fractions related?
- What particular property do they have in common?
- When might it be useful to interchange equivalent fractions?

S: (Students shade, whisper ' 2 '.)
Guide students through the same sequence for a fraction strip folded into eighths.
T : Write the shaded fraction to the right of each fraction strip in your journal.
S: (Students write $\frac{1}{2}, \frac{2}{4}, \frac{4}{8}$.)
T: The unit fractions are different. Discuss with a partner whether or not the fractions are equal, or
equivalent.
S: Since the unit fraction is different, then they are not equal. $\rightarrow$ They have a different number of shaded parts, so I'm not sure. $\rightarrow$ The same amount of the fraction strip is shaded for each one. That must mean they're equal.
T: I hear some uncertainty. Beside our fraction strips, what's another tool we can use to test their equivalence?
S: We can place them on a number line.
T : Let's do that. Place your personal board under the fraction strip folded into halves. Use the fraction strip to measure a number line from 0 to 1 . Label 0 halves, rename the whole, and then label $\frac{1}{2}$.
S: (Measure, draw and label a number line.)
T: Move your personal board down so that your number line is under your fourths fraction strip. On the same number line, label each unit fraction. See if any fractions are located at the same point on the number line.
S: Hey, $\frac{1}{2}$ and $\frac{2}{4}$ are on the same point! $\rightarrow$ So are $\frac{2}{2}, \frac{4}{4}$, and 1. $\rightarrow$ Zeros too, but we already knew that!
T: Discuss with your partner what it might mean when two fractions are at the same point on the number line.
S: I think it means they're the same. $\rightarrow$ It proves what we saw with the fraction strips. They had the same amount shaded before, and now they're in the same place on the number line. $\rightarrow$ The fractions must be equivalent because they are at the same point.
T: I can use the equal sign to show the fractions are equivalent when I write them. (Write $\frac{1}{2}=\frac{2}{4}$.) The equal sign is like a balance. It means 'the same as.' We might read this as $\frac{1}{2}$ is the same as $\frac{2}{4}$ because they have the same value. We just proved that with our number line! As long as the total values on both sides of the equal sign are the same, we can use it to show equivalence. (Write $\frac{2}{2}=\frac{4}{4}=1$.) Turn and tell your partner, is this statement true?
S : The equal sign works when there are two things, not three. $\rightarrow$ But the total value of $\frac{2}{2}$ is 1 , and $\frac{4}{4}$ is 1 , and 1 is 1 , so I think it's true. $\rightarrow$ Remember we can also say 'equals' as 'the same as'? $\frac{2}{2}$ is the same as $\frac{4}{4}$. Those are the same as 1 . They are written differently, but they have the same value.

Direct students to follow the same process to label eighths independently.

T: Fold your last 2 fraction strips. One should be copies of thirds and the other should be copies of sixths. Label the parts with unit fractions and glue these strips

## NOTES ON <br> MULTIPLE MEANS OF ENGAGEMENT:

Slip the Problem Set into a clear plastic sheet protector. Using a dry erase marker, students below grade level can highlight the unit intervals, shade unit fractions as they count, and circle equivalent fractions.
Present an open-ended alternative for students above grade level who may enjoy finding unlimited equivalent fractions for a given point on the number line. Ask (for example), "How many equivalent fractions can you model for 3 halves?"
below the others in your math journal in order from greatest to least. Shade 1 third. Then shade the number of sixths equal to 1 third.
S: (Students follow the process for these 2 strips.)
T: Now work with your partner to measure and draw a new number line using your thirds and sixths. Then using your other strips, find and label all the fractions that are equivalent to thirds and sixths.

Note: If you do not use math journals in your classroom, have your students store these fraction strips in a safe place. They will used again in Lesson 22.

## NOTES ON <br> USING =:

It is worth spending a moment to make sure that students are clear on the meaning of the equal sign in this lesson, as it is an important symbol throughout the topic. Students get into the habit of associating its use with an operation and an answer, not fully understanding its application in a context like $\frac{1}{2}=\frac{2}{4}$.

## Problem Set (10 minutes)

Students should do their personal best to complete the Problem Set within the allotted 10 minutes. For some classes, it may be appropriate to modify the assignment by specifying which problems they work on first. Some problems do not specify a method for solving. Students solve these problems using the RDW approach used for Application Problems.

## Student Debrief (10 minutes)

Lesson Objective: Recognize and show that equivalent fractions refer to the same point on the number line.

The Student Debrief is intended to invite reflection and active processing of the total lesson experience.

Invite students to review their solutions for the Problem Set. They should check work by comparing answers with a partner before going over answers as a class. Look for misconceptions or misunderstandings that can be addressed in the Debrief. Guide students in a conversation to debrief the Problem Set and process the lesson.

You may choose to use any combination of the questions below to lead the discussion.

- The goal of discussion in the debrief is for students to articulate that equivalent fractions refer to the same point on the number line. You might stimulate their thinking by asking them to relate fractions strips to the number line. Make
 sure students are clear on what the word 'equivalent' means and are comfortable using it.
- After students have checked their work for Problems 4 and 5, ask them to use the fraction strips in their math journals to see if they can name another equivalent fraction. ( $\frac{3}{6}$ is the only possibility.) Ask students to talk about how they know the fractions are equivalent and possibly plot them on the same number line to emphasize the lesson objective.
- In anticipation of Lesson 22, guide students to study the fractions in Problem Set Problem 4. Ask them to study the fractions equivalent to wholes. Get students to notice that the number of shaded parts is the same as the total number of parts (numerator and denominator are the same.) Have them use the pattern to name other wholes. Generate excitement by encouraging them to use extremely large numbers, as well as those that are more familiar.


## Exit Ticket (3 minutes)



After the Student Debrief, instruct students to complete the Exit Ticket. A review of their work will help you assess the students' understanding of the concepts that were presented in the lesson today and plan more effectively for future lessons. You may read the questions aloud to the students.

Name $\qquad$ Date $\qquad$

1. Use the unit fractions on the right to count up on the number line. Label the missing fractions.

2. Use the number lines above to:

- Color fractions equal to 1 half blue.
- Color fractions equal to 1 yellow.
- Color fractions equal to 3 halves green.
- Color fractions equal to 2 red.

3. Use the number lines above to make the number sentences true.

$$
\frac{2}{4}=\frac{\square}{6} \quad \frac{6}{6}=\frac{2}{\square}=\frac{\square}{\square} \quad \frac{3}{2}=\frac{\square}{6}=\frac{6}{\square}
$$

4. Jack and Jill use rain gauges the same size and shape to measure rain on the top of a hill. Jack uses a rain gauge marked in fourths of an inch. Jill's gauge measures rain in eighths of an inch. On Thursday, Jack's gauge measured $\frac{2}{4}$ inches of rain. They both had the same amount of water, so what was the reading on Jill's gauge Thursday? Draw a number line to help explain your thinking.
5. Jack and Jill's baby brother Rosco also had a gauge the same size and shape on the same hill. He told Jack and Jill that there had been $\frac{1}{2}$ inch of rain on Thursday. Is he right? Why or why not? Use words and a number line to explain your answer.

Name $\qquad$ Date $\qquad$

1. Claire went home after school and shared with her mother that 1 whole is the same as $\frac{2}{2}$ and $\frac{6}{6}$. Her mother asked why, but she couldn't explain it. Use a number line and words to help Claire show and explain why $1=\frac{2}{2}=\frac{6}{6}$.

Name $\qquad$ Date $\qquad$

1. Use the unit fractions on the right to count up on the number line. Label the missing fractions.

2. Use the number lines above to:

- Color fractions equal to 1 purple.
- Color fractions equal to 2 fourths yellow.
- Color fractions equal to 2 blue.
- Color fractions equal to 5 thirds green.
- Write a pair of fractions that are equivalent.
$\qquad$ $=$ $\qquad$

3. Use the number lines on the previous page to make the number sentences true.

4. Mr. Fairfax ordered 3 large pizzas for a class party. Group A ate $\frac{6}{6}$ of the first pizza, and Group $B$ ate $\frac{8}{6}$ of the second pizza. During the party, the class discussed which group ate more pizzas.
a. Did group A or B eat more pizza? Use words and and pictures to explain your answer to the class.
b. Later Group C ate all remaining slices of pizza. What fraction of the pizza did group C eat? Use words and pictures to explain your answer.
