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# **AP<sup>®</sup> Calculus BC**

## **2016 Free-Response Questions**

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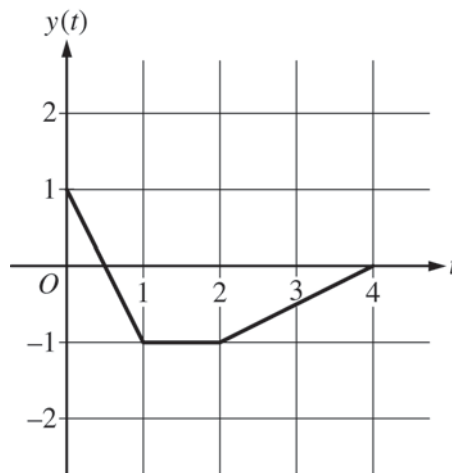
**2016 AP<sup>®</sup> CALCULUS BC FREE-RESPONSE QUESTIONS****CALCULUS BC  
SECTION II, Part A  
Time—30 minutes  
Number of problems—2**

A graphing calculator is required for these problems.

$t$ (hours)	0	1	3	6	8
$R(t)$ (liters / hour)	1340	1190	950	740	700

1. Water is pumped into a tank at a rate modeled by  $W(t) = 2000e^{-t^2/20}$  liters per hour for  $0 \leq t \leq 8$ , where  $t$  is measured in hours. Water is removed from the tank at a rate modeled by  $R(t)$  liters per hour, where  $R$  is differentiable and decreasing on  $0 \leq t \leq 8$ . Selected values of  $R(t)$  are shown in the table above. At time  $t = 0$ , there are 50,000 liters of water in the tank.
- (a) Estimate  $R'(2)$ . Show the work that leads to your answer. Indicate units of measure.
- (b) Use a left Riemann sum with the four subintervals indicated by the table to estimate the total amount of water removed from the tank during the 8 hours. Is this an overestimate or an underestimate of the total amount of water removed? Give a reason for your answer.
- (c) Use your answer from part (b) to find an estimate of the total amount of water in the tank, to the nearest liter, at the end of 8 hours.
- (d) For  $0 \leq t \leq 8$ , is there a time  $t$  when the rate at which water is pumped into the tank is the same as the rate at which water is removed from the tank? Explain why or why not.
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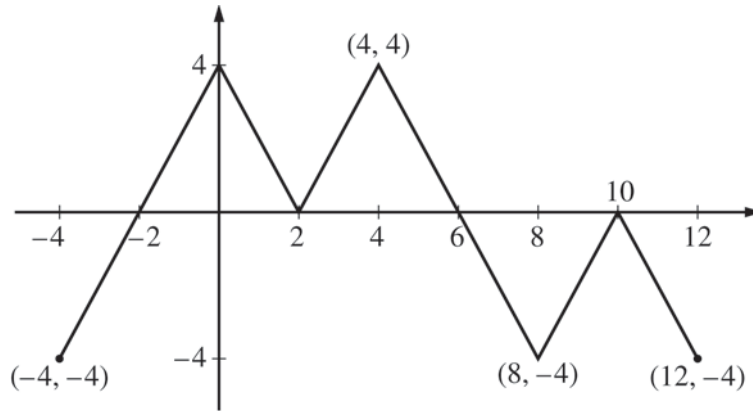
2. At time  $t$ , the position of a particle moving in the  $xy$ -plane is given by the parametric functions  $(x(t), y(t))$ , where  $\frac{dx}{dt} = t^2 + \sin(3t^2)$ . The graph of  $y$ , consisting of three line segments, is shown in the figure above. At  $t = 0$ , the particle is at position  $(5, 1)$ .
- (a) Find the position of the particle at  $t = 3$ .
  - (b) Find the slope of the line tangent to the path of the particle at  $t = 3$ .
  - (c) Find the speed of the particle at  $t = 3$ .
  - (d) Find the total distance traveled by the particle from  $t = 0$  to  $t = 2$ .
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**END OF PART A OF SECTION II**

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**CALCULUS BC**  
**SECTION II, Part B**  
**Time—60 minutes**  
**Number of problems—4**

**No calculator is allowed for these problems.**



Graph of  $f$

3. The figure above shows the graph of the piecewise-linear function  $f$ . For  $-4 \leq x \leq 12$ , the function  $g$  is defined by  $g(x) = \int_2^x f(t) dt$ .
- (a) Does  $g$  have a relative minimum, a relative maximum, or neither at  $x = 10$ ? Justify your answer.
- (b) Does the graph of  $g$  have a point of inflection at  $x = 4$ ? Justify your answer.
- (c) Find the absolute minimum value and the absolute maximum value of  $g$  on the interval  $-4 \leq x \leq 12$ . Justify your answers.
- (d) For  $-4 \leq x \leq 12$ , find all intervals for which  $g(x) \leq 0$ .
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4. Consider the differential equation  $\frac{dy}{dx} = x^2 - \frac{1}{2}y$ .

(a) Find  $\frac{d^2y}{dx^2}$  in terms of  $x$  and  $y$ .

(b) Let  $y = f(x)$  be the particular solution to the given differential equation whose graph passes through the point  $(-2, 8)$ . Does the graph of  $f$  have a relative minimum, a relative maximum, or neither at the point  $(-2, 8)$ ? Justify your answer.

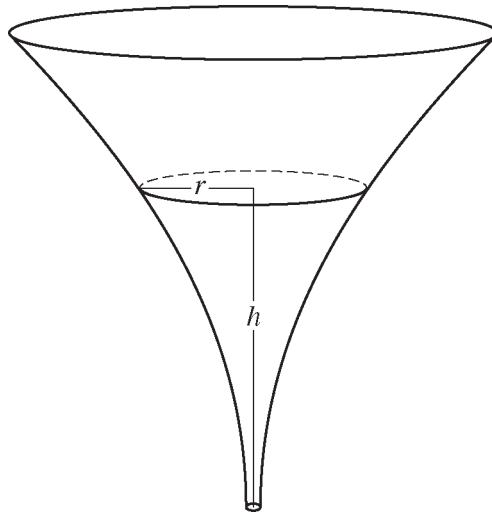
(c) Let  $y = g(x)$  be the particular solution to the given differential equation with  $g(-1) = 2$ . Find

$\lim_{x \rightarrow -1} \left( \frac{g(x) - 2}{3(x + 1)^2} \right)$ . Show the work that leads to your answer.

(d) Let  $y = h(x)$  be the particular solution to the given differential equation with  $h(0) = 2$ . Use Euler's method, starting at  $x = 0$  with two steps of equal size, to approximate  $h(1)$ .

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5. The inside of a funnel of height 10 inches has circular cross sections, as shown in the figure above. At height  $h$ , the radius of the funnel is given by  $r = \frac{1}{20}(3 + h^2)$ , where  $0 \leq h \leq 10$ . The units of  $r$  and  $h$  are inches.
- (a) Find the average value of the radius of the funnel.
- (b) Find the volume of the funnel.
- (c) The funnel contains liquid that is draining from the bottom. At the instant when the height of the liquid is  $h = 3$  inches, the radius of the surface of the liquid is decreasing at a rate of  $\frac{1}{5}$  inch per second. At this instant, what is the rate of change of the height of the liquid with respect to time?
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6. The function  $f$  has a Taylor series about  $x = 1$  that converges to  $f(x)$  for all  $x$  in the interval of convergence. It is known that  $f(1) = 1$ ,  $f'(1) = -\frac{1}{2}$ , and the  $n$ th derivative of  $f$  at  $x = 1$  is given by  $f^{(n)}(1) = (-1)^n \frac{(n-1)!}{2^n}$  for  $n \geq 2$ .
- (a) Write the first four nonzero terms and the general term of the Taylor series for  $f$  about  $x = 1$ .
- (b) The Taylor series for  $f$  about  $x = 1$  has a radius of convergence of 2. Find the interval of convergence. Show the work that leads to your answer.
- (c) The Taylor series for  $f$  about  $x = 1$  can be used to represent  $f(1.2)$  as an alternating series. Use the first three nonzero terms of the alternating series to approximate  $f(1.2)$ .
- (d) Show that the approximation found in part (c) is within 0.001 of the exact value of  $f(1.2)$ .
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**STOP**

**END OF EXAM**