Section I: Multiple-Choice Questions

This is the multiple-choice section of the 2012 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

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AP® Calculus AB Exam

SECTION I: Multiple Choice

2012

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question	Sample Answer
Chicago is a (A) state (B) city (C) country (D) continent (E) village	Ⓐ ● Ⓒ Ⓓ Ⓔ

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

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Minimum 20% post-consumer waste

Form I
Form Code 4IBP4-Q-S
66

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DO NOT seal answer sheet inside

CALCULUS AB

SECTION I, Part A

Time—55 minutes

Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

1. If $y = x\sin x$, then $\dfrac{dy}{dx} =$

(A) $\sin x + \cos x$

(B) $\sin x + x\cos x$

(C) $\sin x - x\cos x$

(D) $x(\sin x + \cos x)$

(E) $x(\sin x - \cos x)$

---

2. Let f be the function given by $f(x) = 300x - x^3$. On which of the following intervals is the function f increasing?

(A) $(-\infty, -10]$ and $[10, \infty)$

(B) $[-10, 10]$

(C) $[0, 10]$ only

(D) $[0, 10\sqrt{3}]$ only

(E) $[0, \infty)$

3. $\int \sec x \tan x \, dx =$

(A) $\sec x + C$

(B) $\tan x + C$

(C) $\dfrac{\sec^2 x}{2} + C$

(D) $\dfrac{\tan^2 x}{2} + C$

(E) $\dfrac{\sec^2 x \tan^2 x}{2} + C$

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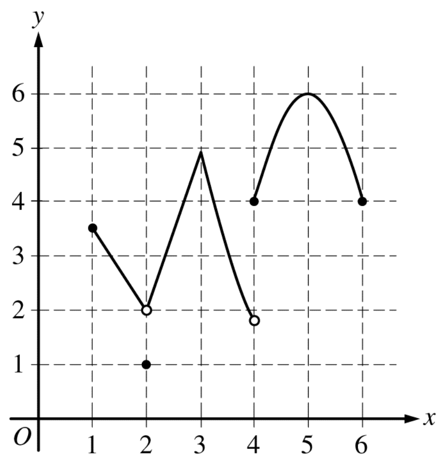
4. If $f(x) = 7x - 3 + \ln x$, then $f'(1) =$

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8



Graph of f

5. The graph of the function f is shown above. Which of the following statements is false?

   (A) $\lim_{x \to 2} f(x)$ exists.

   (B) $\lim_{x \to 3} f(x)$ exists.

   (C) $\lim_{x \to 4} f(x)$ exists.

   (D) $\lim_{x \to 5} f(x)$ exists.

   (E) The function f is continuous at $x = 3$.

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6. A particle moves along the x-axis. The velocity of the particle at time t is $6t - t^2$. What is the total distance traveled by the particle from time $t = 0$ to $t = 3$?

   (A) 3 (B) 6 (C) 9 (D) 18 (E) 27

GO ON TO THE NEXT PAGE.

7. If $y = \left(x^3 - \cos x\right)^5$, then $y' =$

(A) $5\left(x^3 - \cos x\right)^4$

(B) $5\left(3x^2 + \sin x\right)^4$

(C) $5\left(3x^2 + \sin x\right)$

(D) $5\left(3x^2 + \sin x\right)^4 \cdot (6x + \cos x)$

(E) $5\left(x^3 - \cos x\right)^4 \cdot \left(3x^2 + \sin x\right)$

t (hours)	4	7	12	15
$R(t)$ (liters/hour)	6.5	6.2	5.9	5.6

8. A tank contains 50 liters of oil at time $t = 4$ hours. Oil is being pumped into the tank at a rate $R(t)$, where $R(t)$ is measured in liters per hour, and t is measured in hours. Selected values of $R(t)$ are given in the table above. Using a right Riemann sum with three subintervals and data from the table, what is the approximation of the number of liters of oil that are in the tank at time $t = 15$ hours?

(A) 64.9 (B) 68.2 (C) 114.9 (D) 116.6 (E) 118.2

$$f(x) = \begin{cases} \dfrac{(2x+1)(x-2)}{x-2} & \text{for } x \neq 2 \\ k & \text{for } x = 2 \end{cases}$$

9. Let f be the function defined above. For what value of k is f continuous at $x = 2$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

---

10. What is the area of the region in the first quadrant bounded by the graph of $y = e^{x/2}$ and the line $x = 2$?

(A) $2e - 2$ (B) $2e$ (C) $\frac{e}{2} - 1$ (D) $\frac{e-1}{2}$ (E) $e - 1$

11. Let f be the function defined by $f(x) = \sqrt{|x-2|}$ for all x. Which of the following statements is true?

(A) f is continuous but not differentiable at $x = 2$.

(B) f is differentiable at $x = 2$.

(C) f is not continuous at $x = 2$.

(D) $\lim_{x \to 2} f(x) \neq 0$

(E) $x = 2$ is a vertical asymptote of the graph of f.

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12. Using the substitution $u = \sqrt{x}$, $\int_1^4 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx$ is equal to which of the following?

(A) $2\int_1^{16} e^u\, du$ (B) $2\int_1^4 e^u\, du$ (C) $2\int_1^2 e^u\, du$ (D) $\frac{1}{2}\int_1^2 e^u\, du$ (E) $\int_1^4 e^u\, du$

13. The function f is defined by $f(x) = \begin{cases} 2 & \text{for } x < 3 \\ x - 1 & \text{for } x \geq 3. \end{cases}$ What is the value of $\int_1^5 f(x)\,dx$?

(A) 2 (B) 6 (C) 8 (D) 10 (E) 12

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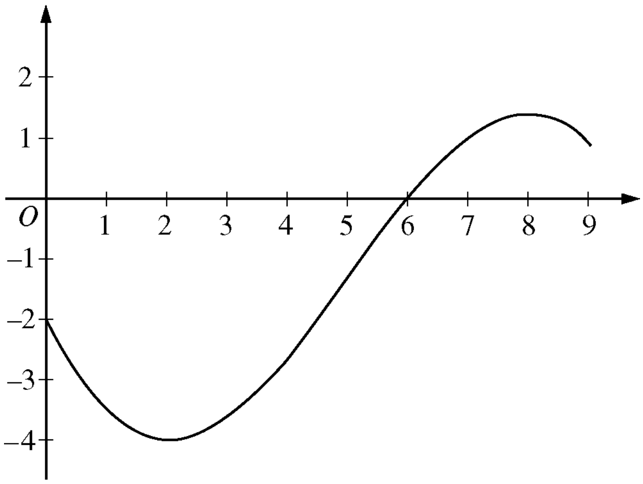
14. If $f(x) = \sqrt{x^2 - 4}$ and $g(x) = 3x - 2$, then the derivative of $f(g(x))$ at $x = 3$ is

(A) $\frac{7}{\sqrt{5}}$ (B) $\frac{14}{\sqrt{5}}$ (C) $\frac{18}{\sqrt{5}}$ (D) $\frac{15}{\sqrt{21}}$ (E) $\frac{30}{\sqrt{21}}$



Graph of f

15. The graph of a differentiable function f is shown above. If $h(x) = \int_0^x f(t)\,dt$, which of the following is true?

(A) $h(6) < h'(6) < h''(6)$

(B) $h(6) < h''(6) < h'(6)$

(C) $h'(6) < h(6) < h''(6)$

(D) $h''(6) < h(6) < h'(6)$

(E) $h''(6) < h'(6) < h(6)$

16. A particle moves along the x-axis with its position at time t given by $x(t) = (t - a)(t - b),$ where a and b are constants and $a \neq b.$ For which of the following values of t is the particle at rest?

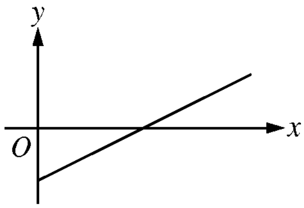
(A) $t = ab$

(B) $t = \frac{a + b}{2}$

(C) $t = a + b$

(D) $t = 2(a + b)$

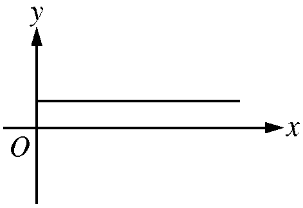
(E) $t = a$ and $t = b$



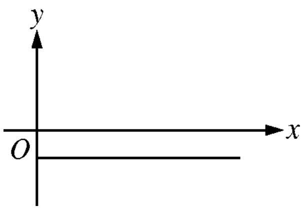
Graph of f

17. The figure above shows the graph of f. If $f(x) = \int_2^x g(t)\,dt$, which of the following could be the graph of $y = g(x)$?

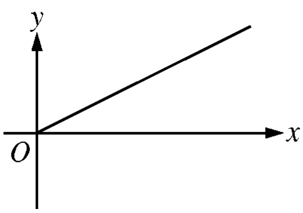
(A)



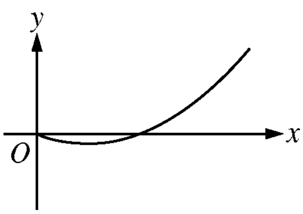
(B)



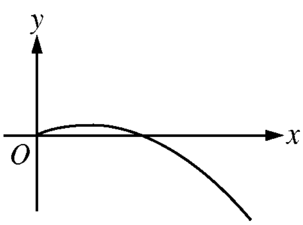
(C)



(D)



(E)



18. $\lim_{h \to 0} \frac{\ln(4+h) - \ln(4)}{h}$ is

(A) 0 (B) $\frac{1}{4}$ (C) 1 (D) e (E) nonexistent

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19. The function f is defined by $f(x) = \frac{x}{x+2}$. What points (x, y) on the graph of f have the property that the line tangent to f at (x, y) has slope $\frac{1}{2}$?

(A) $(0,0)$ only

(B) $\left(\frac{1}{2}, \frac{1}{5}\right)$ only

(C) $(0,0)$ and $(-4,2)$

(D) $(0,0)$ and $\left(4, \frac{2}{3}\right)$

(E) There are no such points.

20. Let $f(x) = (2x+1)^3$ and let g be the inverse function of f. Given that $f(0) = 1$, what is the value of $g'(1)$?

(A) $-\frac{2}{27}$ (B) $\frac{1}{54}$ (C) $\frac{1}{27}$ (D) $\frac{1}{6}$ (E) 6

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21. The line $y = 5$ is a horizontal asymptote to the graph of which of the following functions?

(A) $y = \frac{\sin(5x)}{x}$ (B) $y = 5x$ (C) $y = \frac{1}{x-5}$ (D) $y = \frac{5x}{1-x}$ (E) $y = \frac{20x^2 - x}{1 + 4x^2}$

GO ON TO THE NEXT PAGE.

22. Let f be the function defined by $f(x) = \frac{\ln x}{x}$. What is the absolute maximum value of f?

(A) 1

(B) $\frac{1}{e}$

(C) 0

(D) $-e$

(E) f does not have an absolute maximum value.

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23. If $P(t)$ is the size of a population at time t, which of the following differential equations describes linear growth in the size of the population?

(A) $\frac{dP}{dt} = 200$

(B) $\frac{dP}{dt} = 200t$

(C) $\frac{dP}{dt} = 100t^2$

(D) $\frac{dP}{dt} = 200P$

(E) $\frac{dP}{dt} = 100P^2$

24. Let g be the function given by $g(x) = x^2 e^{kx}$, where k is a constant. For what value of k does g have a critical point at $x = \frac{2}{3}$?

(A) -3 (B) $-\frac{3}{2}$ (C) $-\frac{1}{3}$ (D) 0 (E) There is no such k.

25. Which of the following is the solution to the differential equation $\frac{dy}{dx} = 2\sin x$ with the initial condition $y(\pi) = 1$?

(A) $y = 2\cos x + 3$

(B) $y = 2\cos x - 1$

(C) $y = -2\cos x + 3$

(D) $y = -2\cos x + 1$

(E) $y = -2\cos x - 1$

GO ON TO THE NEXT PAGE.

26. Let g be a function with first derivative given by $g'(x) = \int_0^x e^{-t^2}\, dt$. Which of the following must be true on the interval $0 < x < 2$?

(A) g is increasing, and the graph of g is concave up.

(B) g is increasing, and the graph of g is concave down.

(C) g is decreasing, and the graph of g is concave up.

(D) g is decreasing, and the graph of g is concave down.

(E) g is decreasing, and the graph of g has a point of inflection on $0 < x < 2$.

27. If $(x + 2y) \cdot \frac{dy}{dx} = 2x - y,$ what is the value of $\frac{d^2y}{dx^2}$ at the point $(3, 0)$?

(A) $-\frac{10}{3}$ (B) 0 (C) 2 (D) $\frac{10}{3}$ (E) Undefined

28. For $t \geq 0$, the position of a particle moving along the x-axis is given by $x(t) = \sin t - \cos t$. What is the acceleration of the particle at the point where the velocity is first equal to 0 ?

(A) $-\sqrt{2}$ (B) -1 (C) 0 (D) 1 (E) $\sqrt{2}$

END OF PART A OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART A ONLY.

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB

SECTION I, Part B

Time—50 minutes

Number of questions—17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

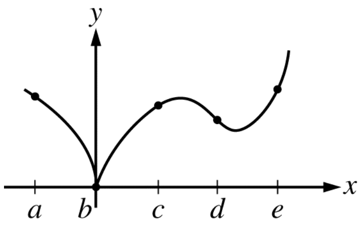
YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).



Graph of f

76. The graph of the function f is shown in the figure above. For which of the following values of x is $f'(x)$ positive and increasing?

(A) a (B) b (C) c (D) d (E) e

GO ON TO THE NEXT PAGE.

77. Let f be a function that is continuous on the closed interval $[2, 4]$ with $f(2) = 10$ and $f(4) = 20$. Which of the following is guaranteed by the Intermediate Value Theorem?

(A) $f(x) = 13$ has at least one solution in the open interval $(2, 4)$.

(B) $f(3) = 15$

(C) f attains a maximum on the open interval $(2, 4)$.

(D) $f'(x) = 5$ has at least one solution in the open interval $(2, 4)$.

(E) $f'(x) > 0$ for all x in the open interval $(2, 4)$.

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78. The graph of $y = e^{\tan x} - 2$ crosses the x-axis at one point in the interval $[0, 1]$. What is the slope of the graph at this point?

(A) 0.606 (B) 2 (C) 2.242 (D) 2.961 (E) 3.747

GO ON TO THE NEXT PAGE.

79. A particle moves along the x-axis. The velocity of the particle at time t is given by $v(t)$, and the acceleration of the particle at time t is given by $a(t)$. Which of the following gives the average velocity of the particle from time $t = 0$ to time $t = 8$?

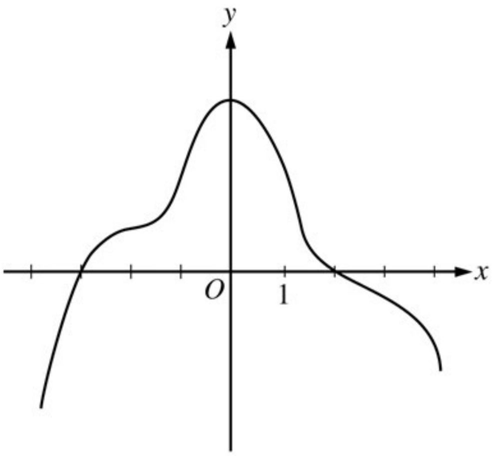
(A) $\dfrac{a(8) - a(0)}{8}$

(B) $\dfrac{1}{8}\int_0^8 v(t)\,dt$

(C) $\dfrac{1}{8}\int_0^8 |v(t)|\,dt$

(D) $\dfrac{1}{2}\int_0^8 v(t)\,dt$

(E) $\dfrac{v(0) + v(8)}{2}$



Graph of f'

80. The graph of f', the derivative of the function f, is shown above. Which of the following statements must be true?

I. f has a relative minimum at $x = -3$.

II. The graph of f has a point of inflection at $x = -2$.

III. The graph of f is concave down for $0 < x < 4$.

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

81. Water is pumped into a tank at a rate of $r(t) = 30\left(1 - e^{-0.16t}\right)$ gallons per minute, where t is the number of minutes since the pump was turned on. If the tank contained 800 gallons of water when the pump was turned on, how much water, to the nearest gallon, is in the tank after 20 minutes?

(A) 380 gallons
(B) 420 gallons
(C) 829 gallons
(D) 1220 gallons
(E) 1376 gallons

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82. If $f'(x) = \sqrt{x^4 + 1} + x^3 - 3x,$ then f has a local maximum at $x =$

(A) -2.314 (B) -1.332 (C) 0.350 (D) 0.829 (E) 1.234

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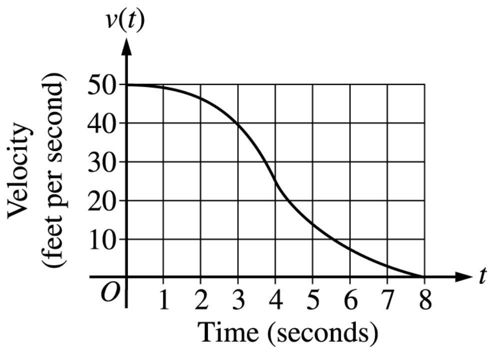

83. The graph above gives the velocity, v, in ft/sec, of a car for $0 \le t \le 8$, where t is the time in seconds. Of the following, which is the best estimate of the distance traveled by the car from $t = 0$ until the car comes to a complete stop?

(A) 21 ft (B) 26 ft (C) 180 ft (D) 210 ft (E) 260 ft

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84. For $-1.5 < x < 1.5$, let f be a function with first derivative given by $f'(x) = e^{\left(x^4 - 2x^2 + 1\right)} - 2$. Which of the following are all intervals on which the graph of f is concave down?

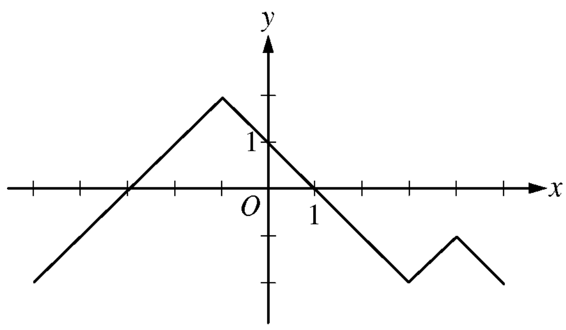
(A) $(-0.418, 0.418)$ only

(B) $(-1, 1)$

(C) $(-1.354, -0.409)$ and $(0.409, 1.354)$

(D) $(-1.5, -1)$ and $(0, 1)$

(E) $(-1.5, -1.354)$, $(-0.409, 0)$, and $(1.354, 1.5)$



Graph of f'

85. The graph of f', the derivative of f, is shown in the figure above. The function f has a local maximum at $x =$

(A) -3 (B) -1 (C) 1 (D) 3 (E) 4

86. If $f'(x) > 0$ for all real numbers x and $\int_4^7 f(t)\,dt = 0$, which of the following could be a table of values for the function f ?

(A)

x	$f(x)$
4	−4
5	−3
7	0

(B)

x	$f(x)$
4	−4
5	−2
7	5

(C)

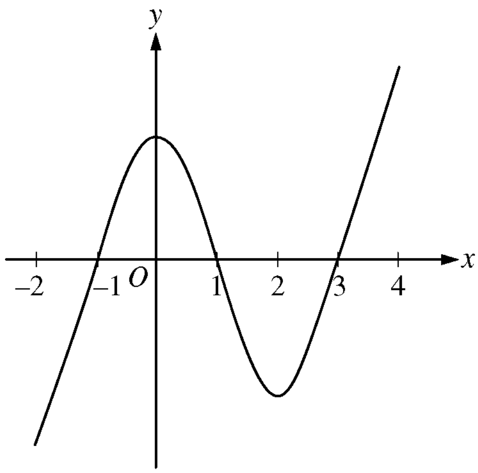
x	$f(x)$
4	−4
5	6
7	3

(D)

x	$f(x)$
4	0
5	0
7	0

(E)

x	$f(x)$
4	0
5	4
7	6



Graph of f''

87. The graph of f'', the second derivative of f, is shown above for $-2 \le x \le 4$. What are all intervals on which the graph of the function f is concave down?

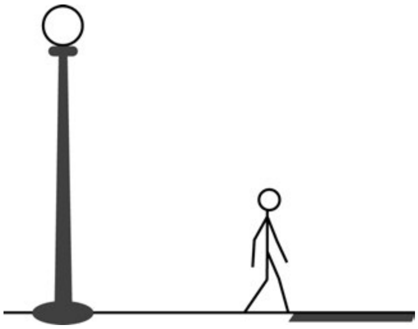
(A) $-1 < x < 1$

(B) $0 < x < 2$

(C) $1 < x < 3$ only

(D) $-2 < x < -1$ only

(E) $-2 < x < -1$ and $1 < x < 3$



88. A person whose height is 6 feet is walking away from the base of a streetlight along a straight path at a rate of 4 feet per second. If the height of the streetlight is 15 feet, what is the rate at which the person's shadow is lengthening?

(A) 1.5 ft/sec (B) 2.667 ft/sec (C) 3.75 ft/sec (D) 6 ft/sec (E) 10 ft/sec

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89. A particle moves along a line so that its acceleration for $t \geq 0$ is given by $a(t) = \frac{t+3}{\sqrt{t^3+1}}$. If the particle's velocity at $t = 0$ is 5, what is the velocity of the particle at $t = 3$?

(A) 0.713 (B) 1.134 (C) 6.134 (D) 6.710 (E) 11.710

GO ON TO THE NEXT PAGE.

90. Let f be a function such that $\int_6^{12} f(2x)\,dx = 10$. Which of the following must be true?

(A) $\int_{12}^{24} f(t)\,dt = 5$

(B) $\int_{12}^{24} f(t)\,dt = 20$

(C) $\int_6^{12} f(t)\,dt = 5$

(D) $\int_6^{12} f(t)\,dt = 20$

(E) $\int_3^{6} f(t)\,dt = 5$

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x	–2	0	3	5	6
$f'(x)$	3	1	4	7	5

91. Let f be a polynomial function with values of $f'(x)$ at selected values of x given in the table above. Which of the following must be true for $-2 < x < 6$?

(A) The graph of f is concave up.

(B) The graph of f has at least two points of inflection.

(C) f is increasing.

(D) f has no critical points.

(E) f has at least two relative extrema.

GO ON TO THE NEXT PAGE.

92. Let R be the region in the first quadrant bounded below by the graph of $y = x^2$ and above by the graph of $y = \sqrt{x}$. R is the base of a solid whose cross sections perpendicular to the x-axis are squares. What is the volume of the solid?

(A) 0.129 (B) 0.300 (C) 0.333 (D) 0.700 (E) 1.271

END OF SECTION I

IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON PART B ONLY.

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**
- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**
- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND ANSWER QUESTIONS 93–96.

GO ON TO THE NEXT PAGE.

Section II: Free-Response Questions

This is the free-response section of the 2012 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

AP® Calculus AB Exam

SECTION II: Free Response

2012

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 30 minutes
Number of Questions
6
Percent of Total Score
50%
Writing Instrument
Either pencil or pen with black or dark blue ink
Weight
The questions are weighted equally, but the parts of a question are not necessarily given equal weight.

Part A

Number of Questions
2
Time
30 minutes
Electronic Device
Graphing calculator required
Percent of Section II Score
33.3%

Part B

Number of Questions
4
Time
60 minutes
Electronic Device
None allowed
Percent of Section II Score
66.6%

IMPORTANT Identification Information

PLEASE PRINT WITH PEN:

1. First two letters of your last name ☐☐

   First letter of your first name ☐

2. Date of birth

   ☐☐ Month ☐☐ Day ☐☐☐☐ Year

3. Six-digit school code

   ☐☐☐☐☐☐

4. Unless I check the box below, I grant the College Board the unlimited right to use, reproduce, and publish my free-response materials, both written and oral, for educational research and instructional purposes. My name and the name of my school will not be used in any way in connection with my free-response materials. I understand that I am free to mark "No" with no effect on my score or its reporting.

   No, I do not grant the College Board these rights. ☐

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.
- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X^2, X, 1, 5).
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.
- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

Form I
Form Code 4IBP-S2

66

CALCULUS AB

SECTION II, Part A

Time—30 minutes

Number of problems—2

A graphing calculator is required for these problems.

GO ON TO THE NEXT PAGE.

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is $55°F$. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\, dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\, dt$ in the context of this problem.

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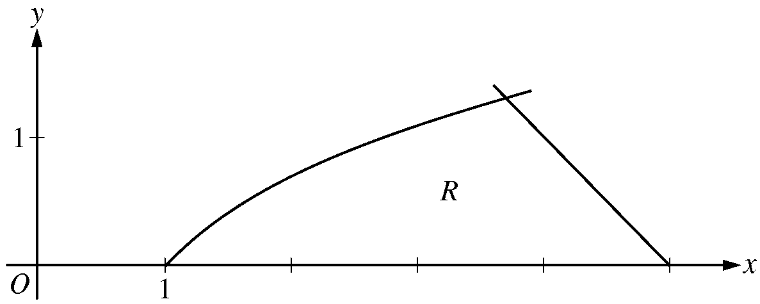
Continue problem 1 on page 5.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\, dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\, dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

GO ON TO THE NEXT PAGE.



2. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.

(a) Find the area of R.

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Continue problem 2 on page 7.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

Do not write beyond this border.

GO ON TO THE NEXT PAGE.

CALCULUS AB

SECTION II, Part B

Time—60 minutes

Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

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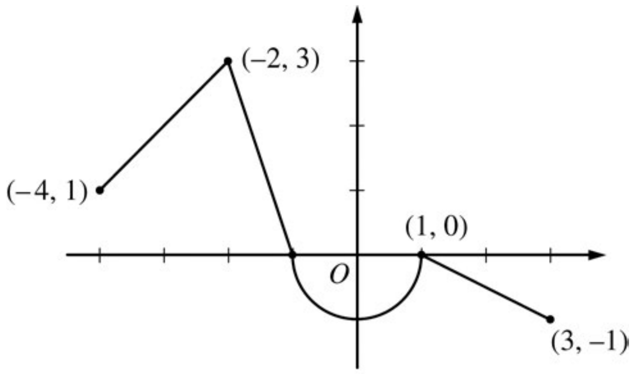

Graph of f

3. Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt$.

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

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Continue problem 3 on page 15.

NO CALCULATOR ALLOWED

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

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GO ON TO THE NEXT PAGE.

NO CALCULATOR ALLOWED

4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

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Continue problem 4 on page 17.

NO CALCULATOR ALLOWED

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\, dx$.

GO ON TO THE NEXT PAGE.

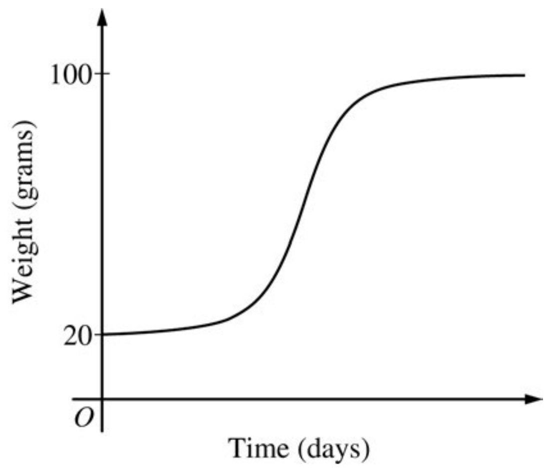
5. The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0,$ when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20.$

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



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Continue problem 5 on page 19.

(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

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Do not write beyond this border.

NO CALCULATOR ALLOWED

6. For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

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Do not write beyond this border.

Continue problem 6 on page 21.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

Do not write beyond this border.

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STOP

END OF EXAM

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT AND BACK COVERS OF THE SECTION II BOOKLET.**
- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**
- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON ALL AP EXAMS YOU HAVE TAKEN THIS YEAR.**

GO ON TO THE NEXT PAGE.

Multiple-Choice Answer Key

The following contains the answers to the multiple-choice questions in this exam.

Answer Key for AP Calculus AB
Practice Exam, Section I

Multiple-Choice Questions	
Question #	**Key**
1	B
2	B
3	A
4	E
5	C
6	D
7	E
8	C
9	E
10	A
11	A
12	C
13	D
14	A
15	A
16	B
17	A
18	B
19	C
20	D
21	E
22	B
23	A
24	A
25	E
26	A
27	A
28	A
76	E
77	A
78	D
79	B
80	E
81	D
82	C
83	D
84	D
85	C
86	B
87	E
88	B
89	E
90	B
91	B
92	A

Free-Response Scoring Guidelines

The following contains the scoring guidelines for the free-response questions in this exam.

AP® CALCULUS AB
2012 SCORING GUIDELINES

Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is $55°$F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

(a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a) $W'(12) \approx \dfrac{W(15) - W(9)}{15 - 9} = \dfrac{67.9 - 61.8}{6}$
$= 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately 1.017 °F per minute at time $t = 12$ minutes.

2 : { 1 : estimate
1 : interpretation with units }

(b) $\int_0^{20} W'(t)\,dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by 16 °F over the interval from $t = 0$ to $t = 20$ minutes.

2 : { 1 : value
1 : interpretation with units }

(c) $\frac{1}{20}\int_0^{20} W(t)\,dt \approx \frac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$
$= \frac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$
$= \frac{1}{20} \cdot 1215.8 = 60.79$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

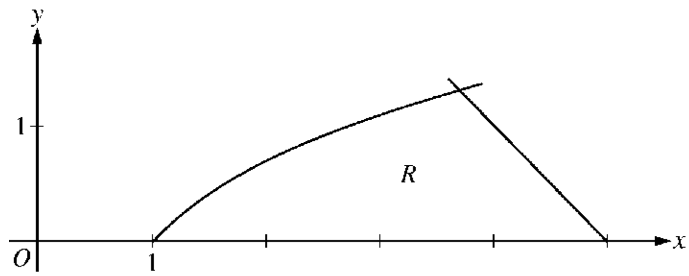
3 : { 1 : left Riemann sum
1 : approximation
1 : underestimate with reason }

(d) $W(25) = 71.0 + \int_{20}^{25} W'(t)\,dt$
$= 71.0 + 2.043155 = 73.043$

2 : { 1 : integral
1 : answer }

Question 2

Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

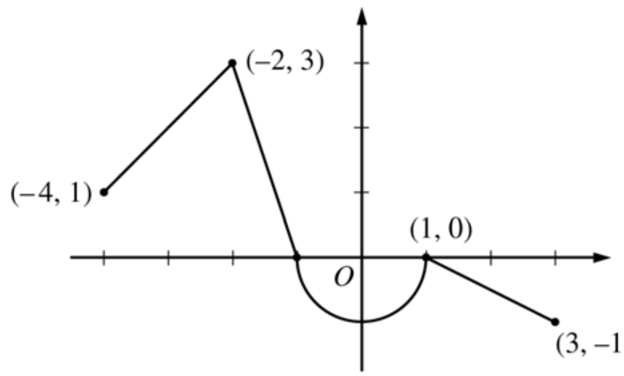
(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$\ln x = 5 - x \;\Rightarrow\; x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) $\text{Area} = \int_0^B \left(5 - y - e^y\right) dy = 2.986 \text{ (or } 2.985)$ OR $\text{Area} = \int_1^A \ln x \, dx + \int_A^5 (5 - x) \, dx = 2.986 \text{ (or } 2.985)$	3 : { 1 : integrand 1 : limits 1 : answer
(b) $\text{Volume} = \int_1^A (\ln x)^2 \, dx + \int_A^5 (5 - x)^2 \, dx$	3 : { 2 : integrands 1 : expression for total volume
(c) $\int_0^k \left(5 - y - e^y\right) dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$	3 : { 1 : integrand 1 : limits 1 : equation

Question 3



Graph of f

Let f be the continuous function defined on $[-4, 3]$ whose graph, consisting of three line segments and a semicircle centered at the origin, is given above. Let g be the function given by $g(x) = \int_1^x f(t)\,dt.$

(a) Find the values of $g(2)$ and $g(-2)$.

(b) For each of $g'(-3)$ and $g''(-3)$, find the value or state that it does not exist.

(c) Find the x-coordinate of each point at which the graph of g has a horizontal tangent line. For each of these points, determine whether g has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(d) For $-4 < x < 3$, find all values of x for which the graph of g has a point of inflection. Explain your reasoning.

(a) $g(2) = \int_1^2 f(t)\,dt = -\frac{1}{2}(1)\left(\frac{1}{2}\right) = -\frac{1}{4}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt$

$= -\left(\frac{3}{2} - \frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{3}{2}$

$2 : \begin{cases} 1 : g(2) \\ 1 : g(-2) \end{cases}$

(b) $g'(x) = f(x) \Rightarrow g'(-3) = f(-3) = 2$
$g''(x) = f'(x) \Rightarrow g''(-3) = f'(-3) = 1$

$2 : \begin{cases} 1 : g'(-3) \\ 1 : g''(-3) \end{cases}$

(c) The graph of g has a horizontal tangent line where $g'(x) = f(x) = 0$. This occurs at $x = -1$ and $x = 1$.

$g'(x)$ changes sign from positive to negative at $x = -1$. Therefore, g has a relative maximum at $x = -1$.

$g'(x)$ does not change sign at $x = 1$. Therefore, g has neither a relative maximum nor a relative minimum at $x = 1$.

$3 : \begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : x = -1 \text{ and } x = 1 \\ 1 : \text{answers with justifications} \end{cases}$

(d) The graph of g has a point of inflection at each of $x = -2$, $x = 0$, and $x = 1$ because $g''(x) = f'(x)$ changes sign at each of these values.

$2 : \begin{cases} 1 : \text{answer} \\ 1 : \text{explanation} \end{cases}$

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2012 SCORING GUIDELINES

Question 4

The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \le x \le 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \le x \le -3 \\ x + 7 & \text{for } -3 < x \le 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2}\,dx$.

(a) $f'(x) = \frac{1}{2}\left(25 - x^2\right)^{-1/2}(-2x) = \frac{-x}{\sqrt{25 - x^2}}, \quad -5 < x < 5$

$2: f'(x)$

(b) $f'(-3) = \frac{3}{\sqrt{25 - 9}} = \frac{3}{4}$

$f(-3) = \sqrt{25 - 9} = 4$

An equation for the tangent line is $y = 4 + \frac{3}{4}(x + 3)$.

$2: \begin{cases} 1: f'(-3) \\ 1: \text{answer} \end{cases}$

(c) $\lim_{x\to -3^-} g(x) = \lim_{x\to -3^-} f(x) = \lim_{x\to -3^-} \sqrt{25 - x^2} = 4$

$\lim_{x\to -3^+} g(x) = \lim_{x\to -3^+} (x + 7) = 4$

Therefore, $\lim_{x\to -3} g(x) = 4$.

$g(-3) = f(-3) = 4$

So, $\lim_{x\to -3} g(x) = g(-3)$.

Therefore, g is continuous at $x = -3$.

$2: \begin{cases} 1: \text{considers one-sided limits} \\ 1: \text{answer with explanation} \end{cases}$

(d) Let $u = 25 - x^2 \Rightarrow du = -2x\,dx$

$$\int_0^5 x\sqrt{25 - x^2}\,dx = -\frac{1}{2}\int_{25}^0 \sqrt{u}\,du$$
$$= \left[-\frac{1}{2}\cdot\frac{2}{3}u^{3/2}\right]_{u=25}^{u=0}$$
$$= -\frac{1}{3}(0 - 125) = \frac{125}{3}$$

$3: \begin{cases} 2: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

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Question 5

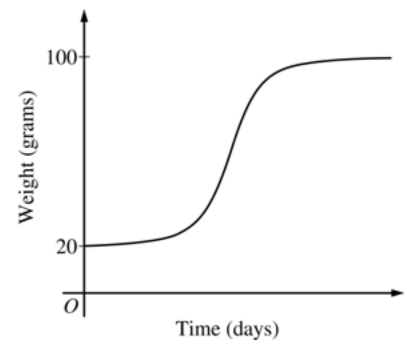
The rate at which a baby bird gains weight is proportional to the difference between its adult weight and its current weight. At time $t = 0$, when the bird is first weighed, its weight is 20 grams. If $B(t)$ is the weight of the bird, in grams, at time t days after it is first weighed, then

$$\frac{dB}{dt} = \frac{1}{5}(100 - B).$$

Let $y = B(t)$ be the solution to the differential equation above with initial condition $B(0) = 20$.

(a) Is the bird gaining weight faster when it weighs 40 grams or when it weighs 70 grams? Explain your reasoning.

(b) Find $\frac{d^2B}{dt^2}$ in terms of B. Use $\frac{d^2B}{dt^2}$ to explain why the graph of B cannot resemble the following graph.



(c) Use separation of variables to find $y = B(t)$, the particular solution to the differential equation with initial condition $B(0) = 20$.

(a) $\left.\frac{dB}{dt}\right|_{B=40} = \frac{1}{5}(60) = 12$

$\left.\frac{dB}{dt}\right|_{B=70} = \frac{1}{5}(30) = 6$

Because $\left.\frac{dB}{dt}\right|_{B=40} > \left.\frac{dB}{dt}\right|_{B=70}$, the bird is gaining weight faster when it weighs 40 grams.

$2 : \begin{cases} 1 : \text{uses } \frac{dB}{dt} \\ 1 : \text{answer with reason} \end{cases}$

(b) $\frac{d^2B}{dt^2} = -\frac{1}{5}\frac{dB}{dt} = -\frac{1}{5}\cdot\frac{1}{5}(100 - B) = -\frac{1}{25}(100 - B)$

Therefore, the graph of B is concave down for $20 \le B < 100$. A portion of the given graph is concave up.

$2 : \begin{cases} 1 : \frac{d^2B}{dt^2} \text{ in terms of } B \\ 1 : \text{explanation} \end{cases}$

(c) $\frac{dB}{dt} = \frac{1}{5}(100 - B)$

$\int \frac{1}{100 - B}\, dB = \int \frac{1}{5}\, dt$

$-\ln|100 - B| = \frac{1}{5}t + C$

Because $20 \le B < 100$, $|100 - B| = 100 - B$.

$-\ln(100 - 20) = \frac{1}{5}(0) + C \Rightarrow -\ln(80) = C$

$100 - B = 80e^{-t/5}$

$B(t) = 100 - 80e^{-t/5}, \quad t \ge 0$

$5 : \begin{cases} 1 : \text{separation of variables} \\ 1 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } B \end{cases}$

Note: max $2/5$ [1-1-0-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

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Question 6

For $0 \le t \le 12$, a particle moves along the x-axis. The velocity of the particle at time t is given by $v(t) = \cos\left(\frac{\pi}{6}t\right)$. The particle is at position $x = -2$ at time $t = 0$.

(a) For $0 \le t \le 12$, when is the particle moving to the left?

(b) Write, but do not evaluate, an integral expression that gives the total distance traveled by the particle from time $t = 0$ to time $t = 6$.

(c) Find the acceleration of the particle at time t. Is the speed of the particle increasing, decreasing, or neither at time $t = 4$? Explain your reasoning.

(d) Find the position of the particle at time $t = 4$.

(a) $v(t) = \cos\left(\frac{\pi}{6}t\right) = 0 \implies t = 3,\ 9$

The particle is moving to the left when $v(t) < 0$.
This occurs when $3 < t < 9$.

$2: \begin{cases} 1: \text{considers } v(t) = 0 \\ 1: \text{interval} \end{cases}$

(b) $\int_0^6 |v(t)|\, dt$

1 : answer

(c) $a(t) = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$a(4) = -\frac{\pi}{6}\sin\left(\frac{2\pi}{3}\right) = -\frac{\sqrt{3}\pi}{12} < 0$

$v(4) = \cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2} < 0$

The speed is increasing at time $t = 4$, because velocity and acceleration have the same sign.

$3: \begin{cases} 1: a(t) \\ 2: \text{conclusion with reason} \end{cases}$

(d)

$$\begin{aligned} x(4) &= -2 + \int_0^4 \cos\left(\frac{\pi}{6}t\right) dt \\ &= -2 + \left[\frac{6}{\pi}\sin\left(\frac{\pi}{6}t\right)\right]_0^4 \\ &= -2 + \frac{6}{\pi}\left[\sin\left(\frac{2\pi}{3}\right) - 0\right] \\ &= -2 + \frac{6}{\pi}\cdot\frac{\sqrt{3}}{2} = -2 + \frac{3\sqrt{3}}{\pi} \end{aligned}$$

$3: \begin{cases} 1: \text{antiderivative} \\ 1: \text{uses initial condition} \\ 1: \text{answer} \end{cases}$

GO ON TO THE NEXT PAGE.

Scoring Worksheet

The following provides a worksheet and conversion table used for calculating a composite score of the exam.

2012 AP Calculus AB Scoring Worksheet

Section I: Multiple Choice

____________ × 1.2000 = ________________
Number Correct (out of 45) — Weighted Section I Score (Do not round)

Section II: Free Response

Question 1 ________ × 1.0000 = ________
(out of 9) (Do not round)

Question 2 ________ × 1.0000 = ________
(out of 9) (Do not round)

Question 3 ________ × 1.0000 = ________
(out of 9) (Do not round)

Question 4 ________ × 1.0000 = ________
(out of 9) (Do not round)

Question 5 ________ × 1.0000 = ________
(out of 9) (Do not round)

Question 6 ________ × 1.0000 = ________
(out of 9) (Do not round)

Sum = ________
Weighted Section II Score (Do not round)

Composite Score

____________ + ____________ = ____________
Weighted Section I Score + Weighted Section II Score = Composite Score (Round to nearest whole number)

AP Score Conversion Chart
Calculus AB

Composite Score Range	AP Score
67-108	5
54-66	4
41-53	3
33-40	2
0-32	1

AP Calculus AB

The College Board

The College Board is a mission-driven not-for-profit organization that connects students to college success and opportunity. Founded in 1900, the College Board was created to expand access to higher education. Today, the membership association is made up of over 6,000 of the world's leading educational institutions and is dedicated to promoting excellence and equity in education. Each year, the College Board helps more than seven million students prepare for a successful transition to college through programs and services in college readiness and college success — including the SAT® and the Advanced Placement Program®. The organization also serves the education community through research and advocacy on behalf of students, educators, and schools. The College Board is committed to the principles of excellence and equity, and that commitment is embodied in all of its programs, services, activities, and concerns.

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