Section I: Multiple-Choice Questions

This is the multiple-choice section of the 2013 AP exam. It includes cover material and other administrative instructions to help familiarize students with the mechanics of the exam. (Note that future exams may differ in look from the following content.)

AP® Calculus AB Exam

SECTION I: Multiple Choice

2013

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO DO SO.

At a Glance

Total Time
1 hour, 45 minutes
Number of Questions
45
Percent of Total Score
50%
Writing Instrument
Pencil required

Part A

Number of Questions
28
Time
55 minutes
Electronic Device
None allowed

Part B

Number of Questions
17
Time
50 minutes
Electronic Device
Graphing calculator required

Instructions

Section I of this exam contains 45 multiple-choice questions and 4 survey questions. For Part A, fill in only the circles for numbers 1 through 28 on page 2 of the answer sheet. For Part B, fill in only the circles for numbers 76 through 92 on page 3 of the answer sheet. The survey questions are numbers 93 through 96.

Indicate all of your answers to the multiple-choice questions on the answer sheet. No credit will be given for anything written in this exam booklet, but you may use the booklet for notes or scratch work. After you have decided which of the suggested answers is best, completely fill in the corresponding circle on the answer sheet. Give only one answer to each question. If you change an answer, be sure that the previous mark is erased completely. Here is a sample question and answer.

Sample Question   Sample Answer

Chicago is a   Ⓐ ● Ⓒ Ⓓ Ⓔ
(A) state
(B) city
(C) country
(D) continent
(E) village

Use your time effectively, working as quickly as you can without losing accuracy. Do not spend too much time on any one question. Go on to other questions and come back to the ones you have not answered if you have time. It is not expected that everyone will know the answers to all of the multiple-choice questions.

Your total score on the multiple-choice section is based only on the number of questions answered correctly. Points are not deducted for incorrect answers or unanswered questions.

Form I
Form Code 4JBP4-Q-S

66

CALCULUS AB
SECTION I, Part A
Time—55 minutes
Number of questions—28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

In this exam:

(1)   Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2)   The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

GO ON TO THE NEXT PAGE.

1. $\displaystyle\lim_{x\to 2}\frac{x^2 + x - 6}{x^2 - 4}$ is

(A) $-\dfrac{1}{4}$ (B) 0 (C) 1 (D) $\dfrac{5}{4}$ (E) nonexistent

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2. If $f(x) = x^3 - x^2 + x - 1,$ then $f'(2) =$

(A) 10 (B) 9 (C) 7 (D) 5 (E) 3

GO ON TO THE NEXT PAGE.

3. Which of the following definite integrals has the same value as $\int_0^4 xe^{x^2}\,dx$?

(A) $\dfrac{1}{2}\int_0^4 e^u\,du$

(B) $\dfrac{1}{2}\int_0^{16} e^u\,du$

(C) $2\int_0^2 e^u\,du$

(D) $2\int_0^4 e^u\,du$

(E) $2\int_0^{16} e^u\,du$

GO ON TO THE NEXT PAGE.

4. Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$?

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\dfrac{7}{3}(x - 1)$

(C) $y + 3 = \dfrac{1}{3}(x - 1)$

(D) $y + 3 = \dfrac{7}{3}(x - 1)$

(E) $y + 3 = \dfrac{11}{3}(x - 1)$

GO ON TO THE NEXT PAGE.

5. If g is the function given by $g(x) = \frac{1}{3}x^3 + \frac{3}{2}x^2 - 70x + 5$, on which of the following intervals is g decreasing?

(A) $(-\infty, -10)$ and $(7, \infty)$

(B) $(-\infty, -7)$ and $(10, \infty)$

(C) $(-\infty, 10)$

(D) $(-10, 7)$

(E) $(-7, 10)$

6. $\displaystyle\int_2^4 \frac{dx}{5 - 3x} =$

(A) $-\ln 7$ (B) $-\dfrac{\ln 7}{3}$ (C) $\dfrac{\ln 7}{3}$ (D) $\ln 7$ (E) $3\ln 7$

GO ON TO THE NEXT PAGE.

7. Let f be the function given by $f(x) = x^3 - 6x^2 + 8x - 2$. What is the instantaneous rate of change of f at $x = 3$?

(A) -5 (B) $-\dfrac{15}{4}$ (C) -1 (D) 6 (E) 17
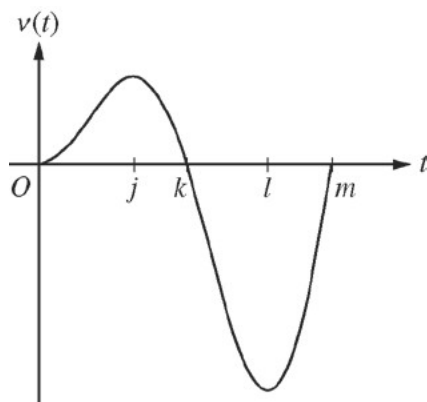
8. A particle moves along a straight line. The graph of the particle's velocity $v(t)$ at time t is shown above for $0 \le t \le m$, where j, k, l, and m are constants. The graph intersects the horizontal axis at $t = 0$, $t = k$, and $t = m$ and has horizontal tangents at $t = j$ and $t = l$. For what values of t is the speed of the particle decreasing?

(A) $j \le t \le l$

(B) $k \le t \le m$

(C) $j \le t \le k$ and $l \le t \le m$

(D) $0 \le t \le j$ and $k \le t \le l$

(E) $0 \le t \le j$ and $l \le t \le m$

GO ON TO THE NEXT PAGE.

9. Let f be the function given by $f(x) = \dfrac{(x-2)^2(x+3)}{(x-2)(x+1)}$. For which of the following values of x is f not continuous?

(A) -3 and -1 only

(B) -3, -1, and 2

(C) -1 only

(D) -1 and 2 only

(E) 2 only

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10. A particle moves along the x-axis with velocity given by $v(t) = 3t^2 - 4$ for time $t \geq 0$. If the particle is at position $x = -2$ at time $t = 0$, what is the position of the particle at time $t = 3$?

(A) 13   (B) 15   (C) 16   (D) 17   (E) 25

GO ON TO THE NEXT PAGE.

11. Let f be the function defined by $f(x) = \int_0^x \left(2t^3 - 15t^2 + 36t\right) dt$. On which of the following intervals is the graph of $y = f(x)$ concave down?

(A) $(-\infty, 0)$ only

(B) $(-\infty, 2)$

(C) $(0, \infty)$

(D) $(2, 3)$ only

(E) $(3, \infty)$ only

GO ON TO THE NEXT PAGE.

12. For which of the following does $\lim\limits_{x \to \infty} f(x) = 0$?

    I. $f(x) = \dfrac{\ln x}{x^{99}}$

    II. $f(x) = \dfrac{e^x}{\ln x}$

    III. $f(x) = \dfrac{x^{99}}{e^x}$

(A) I only

(B) II only

(C) III only

(D) I and II only

(E) I and III only

GO ON TO THE NEXT PAGE.

13. Let f be a differentiable function such that $f(0) = -5$ and $f'(x) \le 3$ for all x. Of the following, which is not a possible value for $f(2)$?

(A) -10 (B) -5 (C) 0 (D) 1 (E) 2

$$f(x) = \begin{cases} x + b & \text{if } x \le 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

14. Let f be the function given above. What are all values of a and b for which f is differentiable at $x = 1$?

(A) $a = \dfrac{1}{2}$ and $b = -\dfrac{1}{2}$

(B) $a = \dfrac{1}{2}$ and $b = \dfrac{3}{2}$

(C) $a = \dfrac{1}{2}$ and b is any real number

(D) $a = b + 1$, where b is any real number

(E) There are no such values of a and b.

GO ON TO THE NEXT PAGE.

$f(3)$	$g(3)$	$f'(3)$	$g'(3)$
-1	2	5	-2

15. The table above gives values for the functions f and g and their derivatives at $x = 3$. Let k be the function given by $k(x) = \dfrac{f(x)}{g(x)}$, where $g(x) \neq 0$. What is the value of $k'(3)$?

(A) $-\dfrac{5}{2}$ (B) -2 (C) 2 (D) 3 (E) 8

16. If $y = 5x\sqrt{x^2 + 1}$, then $\dfrac{dy}{dx}$ at $x = 3$ is

(A) $\dfrac{5}{2\sqrt{10}}$ (B) $\dfrac{15}{\sqrt{10}}$ (C) $\dfrac{15}{2\sqrt{10}} + 5\sqrt{10}$ (D) $\dfrac{45}{\sqrt{10}} + 5\sqrt{10}$ (E) $\dfrac{45}{\sqrt{10}} + 15\sqrt{10}$

GO ON TO THE NEXT PAGE.

17. If $\lim\limits_{h \to 0} \dfrac{\arcsin(a+h) - \arcsin(a)}{h} = 2,$ which of the following could be the value of a?

(A) $\dfrac{\sqrt{2}}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\sqrt{3}$ (D) $\dfrac{1}{2}$ (E) 2

18. If $\ln(2x + y) = x + 1,$ then $\dfrac{dy}{dx} =$

(A) -2 (B) $2x + y - 2$ (C) $2x + y$ (D) $4x + 2y - 2$ (E) $y - \dfrac{y}{x}$

19. The figure above shows the graph of the function g and the line tangent to the graph of g at $x = -1$. Let h be the function given by $h(x) = e^x \cdot g(x)$. What is the value of $h'(-1)$?

(A) $\dfrac{9}{e}$ (B) $\dfrac{-3}{e}$ (C) $\dfrac{-6}{e}$ (D) $\dfrac{-6}{e} - \dfrac{3}{e^2}$ (E) -6

GO ON TO THE NEXT PAGE.

20. For $x > 0$, $\dfrac{d}{dx}\left(\displaystyle\int_0^{2x} \ln\left(t^3 + 1\right) dt\right) =$

(A) $\ln\left(x^3 + 1\right)$

(B) $\ln\left(8x^3 + 1\right)$

(C) $2\ln\left(x^3 + 1\right)$

(D) $2\ln\left(8x^3 + 1\right)$

(E) $24x^2 \ln\left(8x^3 + 1\right)$

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(2, 2) (4, 2)

(7, −2)
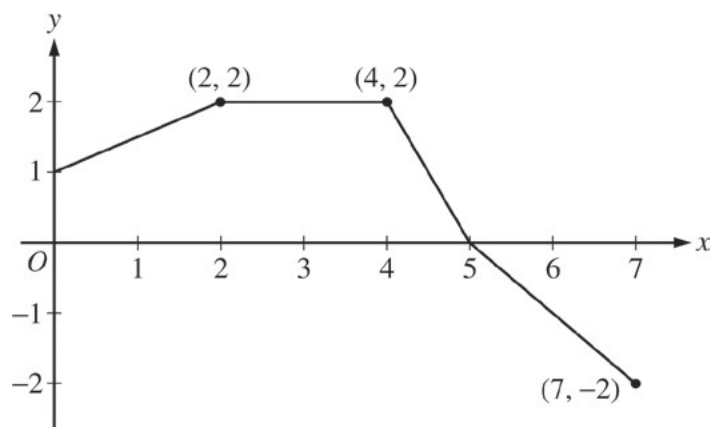
Graph of f

21. The graph of a function f is shown above. What is the value of $\int_0^7 f(x)\,dx$?

(A) 6   (B) 8   (C) 10   (D) 14   (E) 18

GO ON TO THE NEXT PAGE.

22. The function f is continuous for all real numbers, and the average rate of change of f on the closed interval $[6, 9]$ is $-\dfrac{3}{2}$. For $6 < c < 9$, there is no value of c such that $f'(c) = -\dfrac{3}{2}$. Of the following, which must be true?

(A) $\dfrac{1}{3}\displaystyle\int_6^9 f(x)\,dx = -\dfrac{3}{2}$

(B) $\displaystyle\int_6^9 f(x)\,dx$ does not exist.

(C) $\dfrac{f'(6) + f'(9)}{2} = -\dfrac{3}{2}$

(D) $f'(x) < 0$ for all x in the open interval $(6, 9)$.

(E) f is not differentiable on the open interval $(6, 9)$.

23. Let f be the function defined by $f(x) = 2x + e^x$. If $g(x) = f^{-1}(x)$ for all x and the point $(0, 1)$ is on the graph of f, what is the value of $g'(1)$?

(A) $\dfrac{1}{2 + e}$ (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) 3   (E) $2 + e$

GO ON TO THE NEXT PAGE.

24. The function g is given by $g(x) = 4x^3 + 3x^2 - 6x + 1$. What is the absolute minimum value of g on the closed interval $[-2, 1]$?

(A) -7 (B) $-\dfrac{3}{4}$ (C) 0 (D) 2 (E) 6

25. Which of the following is the solution to the differential equation $\dfrac{dy}{dx} = e^{y+x}$ with the initial condition $y(0) = -\ln 4$?

(A) $y = -x - \ln 4$

(B) $y = x - \ln 4$

(C) $y = -\ln\left(-e^x + 5\right)$

(D) $y = -\ln\left(e^x + 3\right)$

(E) $y = \ln\left(e^x + 3\right)$

GO ON TO THE NEXT PAGE.

26. Which of the following is an antiderivative of $f(x) = \sqrt{1 + x^3}$?

(A) $\frac{2}{3}\left(1 + x^3\right)^{3/2}$

(B) $\dfrac{\frac{2}{3}\left(1 + x^3\right)^{3/2}}{3x^2}$

(C) $\displaystyle\int_0^{1+x^3} \sqrt{t}\ dt$

(D) $\displaystyle\int_0^{x^3} \sqrt{1 + t}\ dt$

(E) $\displaystyle\int_0^x \sqrt{1 + t^3}\ dt$

27. For time $t \geq 0$, the height h of an object suspended from a spring is given by $h(t) = 16 + 7\cos\left(\frac{\pi t}{4}\right)$. What is the average height of the object from $t = 0$ to $t = 2$?

(A) 16   (B) $\dfrac{39}{2}$ (C) $16 - \dfrac{14}{\pi}$ (D) $16 + \dfrac{14}{\pi}$ (E) $32 + \dfrac{28}{\pi}$

GO ON TO THE NEXT PAGE.

28. The function f is defined by $f(x) = \sin x + \cos x$ for $0 \le x \le 2\pi$. What is the x-coordinate of the point of inflection where the graph of f changes from concave down to concave up?

(A) $\dfrac{\pi}{4}$ (B) $\dfrac{3\pi}{4}$ (C) $\dfrac{5\pi}{4}$ (D) $\dfrac{7\pi}{4}$ (E) $\dfrac{9\pi}{4}$

END OF PART A OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART A ONLY.**

DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB
SECTION I, Part B
Time — 50 minutes
Number of questions — 17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON
THIS PART OF THE EXAM.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding circle on the answer sheet. No credit will be given for anything written in the exam book. Do not spend too much time on any one problem.

BE SURE YOU ARE USING PAGE 3 OF THE ANSWER SHEET TO RECORD YOUR ANSWERS TO QUESTIONS NUMBERED 76–92.

YOU MAY NOT RETURN TO PAGE 2 OF THE ANSWER SHEET.

In this exam:

(1)   The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2)   Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3)   The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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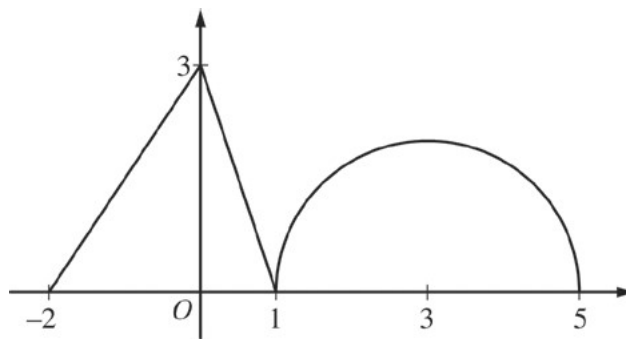
Graph of f

76. The graph of the function f shown above consists of two line segments and a semicircle. Let g be defined by $g(x) = \int_0^x f(t)\, dt$. What is the value of $g(5)$?

(A) 0   (B) $-1.5 + 2\pi$ (C) 2π (D) $1.5 + 2\pi$ (E) $4.5 + 2\pi$

GO ON TO THE NEXT PAGE.

77. The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere?

(The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(A) 0.141 cm   (B) 0.244 cm   (C) 0.250 cm   (D) 0.489 cm   (E) 0.977 cm

78. Let f and g be continuous functions such that $\int_0^{10} f(x)\, dx = 21$, $\int_0^{10} \frac{1}{2} g(x)\, dx = 8$, and $\int_3^{10} (f(x) - g(x))\, dx = 2$. What is the value of $\int_0^3 (f(x) - g(x))\, dx$?

(A) 3   (B) 7   (C) 11   (D) 15   (E) 19

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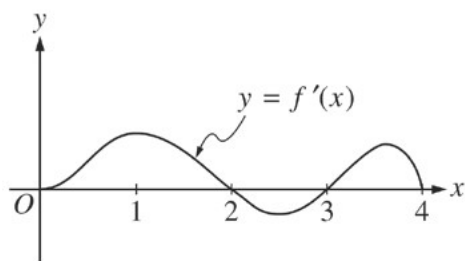
79. The figure above shows the graph of f', the derivative of the function f. If $f(0) = 0$, which of the following could be the graph of f?

(A)

(B)

(C)

(D)

(E)

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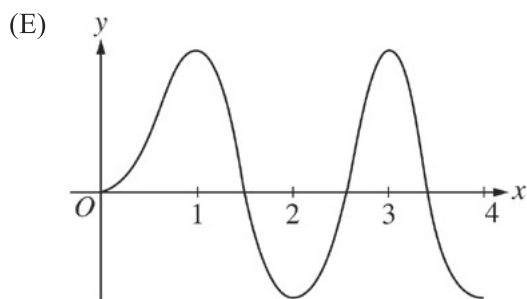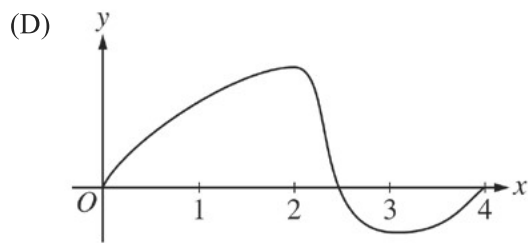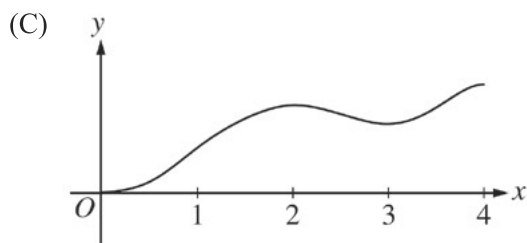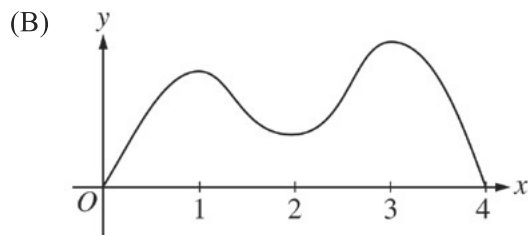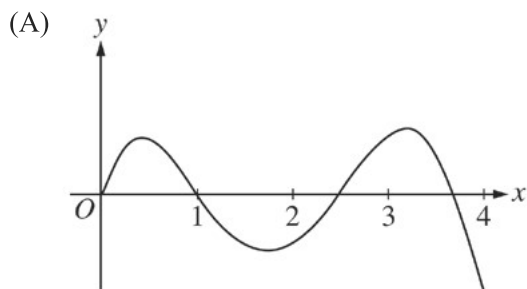
80. For time $t \geq 0,$ the position of a particle traveling along a line is given by a differentiable function $s.$ If s is increasing for $0 \leq t < 2$ and s is decreasing for $t > 2,$ which of the following is the total distance the particle travels for $0 \leq t \leq 5$?

(A) $s(0) + \int_0^2 s'(t)\,dt - \int_2^5 s'(t)\,dt$

(B) $s(0) + \int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$

(C) $\int_2^5 s'(t)\,dt - \int_0^2 s'(t)\,dt$

(D) $\left| \int_0^5 s'(t)\,dt \right|$

(E) $\int_0^5 |s'(t)|\,dt$

GO ON TO THE NEXT PAGE.

81. A cup of tea is cooling in a room that has a constant temperature of 70 degrees Fahrenheit $(°F)$. If the initial temperature of the tea, at time $t = 0$ minutes, is $200°F$ and the temperature of the tea changes at the rate $R(t) = -6.89e^{-0.053t}$ degrees Fahrenheit per minute, what is the temperature, to the nearest degree, of the tea after 4 minutes?

(A) $175°F$ (B) $130°F$ (C) $95°F$ (D) $70°F$ (E) $45°F$

GO ON TO THE NEXT PAGE.

82. The derivative of the function f is given by $f'(x) = x^3 - 4\sin\left(x^2\right) + 1$. On the interval $(-2.5, 2.5)$, at which of the following values of x does f have a relative maximum?

   (A) -1.970 and 0

   (B) -1.467 and 1.075

   (C) -0.475, 0.542, and 1.396

   (D) -0.475 and 1.396 only

   (E) 0.542 only

x	0	0.5	1	1.5	2	2.5	3
$f(x)$	0	4	10	18	28	40	54

83. The table above gives selected values for a continuous function f. If f is increasing over the closed interval $[0, 3]$, which of the following could be the value of $\int_0^3 f(x)\,dx$?

   (A) 50   (B) 62   (C) 77   (D) 100   (E) 154
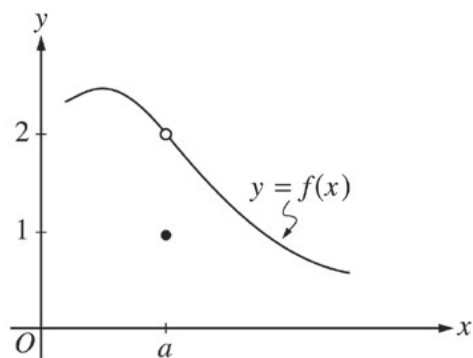
84. The graph of a function f is shown in the figure above. Which of the following statements is true?

(A) $f(a) = 2$

(B) f is continuous at $x = a$.

(C) $\lim\limits_{x \to a} f(x) = 1$

(D) $\lim\limits_{x \to a} f(x) = 2$

(E) $\lim\limits_{x \to a} f(x)$ does not exist.

85. A particle moves along the x-axis so that at time $t \geq 0$ its position is given by $x(t) = \cos \sqrt{t}$. What is the velocity of the particle at the first instance the particle is at the origin?

(A) -1 (B) -0.624 (C) -0.318 (D) 0 (E) 0.065

GO ON TO THE NEXT PAGE.

86. If $f'(x) > 0$ for all x and $f''(x) < 0$ for all x, which of the following could be a table of values for f?

(A)

x	$f(x)$
-1	4
0	3
1	1

(B)

x	$f(x)$
-1	4
0	4
1	4

(C)

x	$f(x)$
-1	4
0	5
1	6

(D)

x	$f(x)$
-1	4
0	5
1	7

(E)

x	$f(x)$
-1	4
0	6
1	7

GO ON TO THE NEXT PAGE.

87. Let f be the function with first derivative given by $f'(x) = \left(3 - 2x - x^2\right)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

(A) Two   (B) Three   (C) Four   (D) Five   (E) Six

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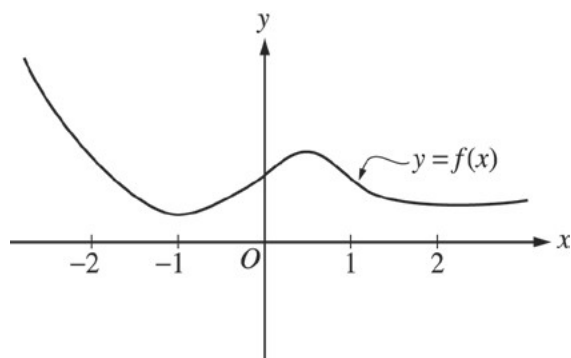
88. The graph of a twice-differentiable function f is shown in the figure above. Which of the following is true?

(A) $f'(-1) < f'(1) < f'(0)$

(B) $f'(-1) < f'(0) < f'(1)$

(C) $f'(0) < f'(-1) < f'(1)$

(D) $f'(1) < f'(-1) < f'(0)$

(E) $f'(1) < f'(0) < f'(-1)$

89. What is the volume of the solid generated when the region bounded by the graph of $x = \sqrt{y-2}$ and the lines $x = 0$ and $y = 5$ is revolved about the y-axis?

(A) 3.464   (B) 4.500   (C) 7.854   (D) 10.883   (E) 14.137

90. The population P of a city grows according to the differential equation $\dfrac{dP}{dt} = kP$, where k is a constant and t is measured in years. If the population of the city doubles every 12 years, what is the value of k?

(A) 0.058   (B) 0.061   (C) 0.167   (D) 0.693   (E) 8.318

91. The function f is continuous and $\int_0^8 f(u)\,du = 6$. What is the value of $\int_1^3 xf\left(x^2 - 1\right)dx$?

(A) $\dfrac{3}{2}$ (B) 3   (C) 6   (D) 12   (E) 24

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92. The function f is defined for all x in the closed interval $[a, b]$. If f does not attain a maximum value on $[a, b]$, which of the following must be true?

(A) f is not continuous on $[a, b]$.

(B) f is not bounded on $[a, b]$.

(C) f does not attain a minimum value on $[a, b]$.

(D) The graph of f has a vertical asymptote in the interval $[a, b]$.

(E) The equation $f'(x) = 0$ does not have a solution in the interval $[a, b]$.

GO ON TO THE NEXT PAGE.

END OF SECTION I

**IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY
CHECK YOUR WORK ON PART B ONLY.**

DO NOT GO ON TO SECTION II UNTIL YOU ARE TOLD TO DO SO.
_____

MAKE SURE YOU HAVE DONE THE FOLLOWING.

- **PLACED YOUR AP NUMBER LABEL ON YOUR ANSWER SHEET**

- **WRITTEN AND GRIDDED YOUR AP NUMBER CORRECTLY ON YOUR ANSWER SHEET**

- **TAKEN THE AP EXAM LABEL FROM THE FRONT OF THIS BOOKLET AND PLACED IT ON YOUR ANSWER SHEET**

**AFTER TIME HAS BEEN CALLED, TURN TO PAGE 38 AND
ANSWER QUESTIONS 93–96.**

Section II: Free-Response Questions

AP® Calculus AB Exam

SECTION II: Free Response

2013

DO NOT OPEN THIS BOOKLET OR BREAK THE SEALS ON PART B UNTIL YOU ARE TOLD TO DO SO.

Instructions

The questions for Section II are printed in this booklet. Do not break the seals on Part B until you are told to do so. Write your solution to each part of each question in the space provided. Write clearly and legibly. Cross out any errors you make; erased or crossed-out work will not be scored.

Manage your time carefully. During the timed portion for Part A, work only on the questions in Part A. You are permitted to use your calculator to solve an equation, find the derivative of a function at a point, or calculate the value of a definite integral. However, you must clearly indicate the setup of your question, namely the equation, function, or integral you are using. If you use other built-in features or programs, you must show the mathematical steps necessary to produce your results. During the timed portion for Part B, you may continue to work on the questions in Part A without the use of a calculator.

For each part of Section II, you may wish to look over the questions before starting to work on them. It is not expected that everyone will be able to complete all parts of all questions.

- Show all of your work. Clearly label any functions, graphs, tables, or other objects that you use. Your work will be scored on the correctness and completeness of your methods as well as your answers. Answers without supporting work will usually not receive credit. Justifications require that you give mathematical (noncalculator) reasons.

- Your work must be expressed in standard mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as fnInt(X², X, 1, 5).

- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If you use decimal approximations in calculations, your work will be scored on accuracy. Unless otherwise specified, your final answers should be accurate to three places after the decimal point.

- Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.
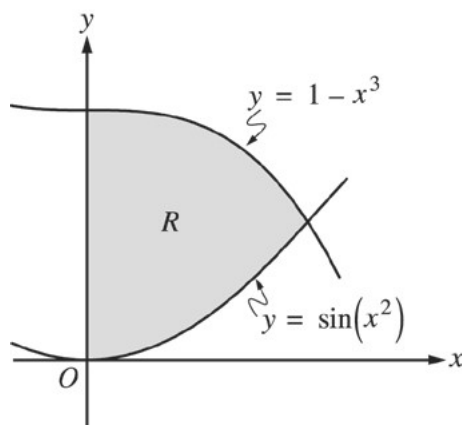
CALCULUS AB
SECTION II, Part A
Time—30 minutes
Number of problems—2

A graphing calculator is required for these problems.

1. Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin(x^2)$, as shown in the figure above.

(a) Find the area of R.

(b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin\left(x^2\right)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

(c) Find the volume of the solid generated when R is revolved about the line $y = -3$.

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2. The penguin population on an island is modeled by a differentiable function P of time t, where $P(t)$ is the number of penguins and t is measured in years, for $0 \leq t \leq 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

$$B(t) = 1000e^{0.06t} \text{ penguins per year}$$

and the death rate for the penguins on the island is modeled by

$$D(t) = 250e^{0.1t} \text{ penguins per year.}$$

(a) What is the rate of change of the penguin population on the island at time $t = 0$?

(b) To the nearest whole number, what is the penguin population on the island at time $t = 40$?

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(c) To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \leq t \leq 40$?

(d) To the nearest whole number, find the absolute minimum penguin population and the absolute maximum penguin population on the island for $0 \leq t \leq 40$. Show the analysis that leads to your answers.

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END OF PART A OF SECTION II
IF YOU FINISH BEFORE TIME IS CALLED, YOU MAY CHECK YOUR WORK ON
PART A ONLY. DO NOT GO ON TO PART B UNTIL YOU ARE TOLD TO DO SO.

CALCULUS AB
SECTION II, Part B
Time—60 minutes
Number of problems—4

No calculator is allowed for these problems.

DO NOT BREAK THE SEALS UNTIL YOU ARE TOLD TO DO SO.

NO CALCULATOR ALLOWED

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

3. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\, dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7 \text{ GL}/\text{day}$.

(d) The equation $A = 0.3 W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

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4. Let f be the function given by $f(x) = \left(x^2 - 2x - 1\right)e^x$.

   (a) Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

   $\lim_{x \to \infty} f(x) = $ _____

   $\lim_{x \to -\infty} f(x) = $ _____

   (b) Find the intervals on which f is increasing. Show the analysis that leads to your answer.

(c) Find the intervals on which the graph of f is concave down. Show the analysis that leads to your answer.

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Graph of f

5. The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above.

   Let g be the function given by $g(x) = \int_{-2}^{x} f(t)\, dt$.

   (a) Find $g(-6)$ and $g(3)$.

   (b) Find $g'(0)$.

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(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.

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6. Let f be a function with $f(2) = -8$ such that for all points (x, y) on the graph of f, the slope is given by $\dfrac{3x^2}{y}$.

   (a) Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.

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(b) Find an expression for $y = f(x)$ by solving the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{y}$ with the initial condition

$f(2) = -8.$

STOP

END OF EXAM

_____

THE FOLLOWING INSTRUCTIONS APPLY TO THE COVERS OF THE SECTION II BOOKLET.

- **MAKE SURE YOU HAVE COMPLETED THE IDENTIFICATION INFORMATION AS REQUESTED ON THE FRONT <u>AND</u> BACK COVERS OF THE SECTION II BOOKLET.**

- **CHECK TO SEE THAT YOUR AP NUMBER LABEL APPEARS IN THE BOX ON THE COVER.**

- **MAKE SURE YOU HAVE USED THE SAME SET OF AP NUMBER LABELS ON <u>ALL</u> AP EXAMS YOU HAVE TAKEN THIS YEAR.**

Multiple-Choice Answer Key

The following contains the answers to the
multiple-choice questions in this exam.

Answer Key for AP Calculus AB Practice Exam, Section I

Question 1: D

Question 2: B

Question 3: B

Question 4: E

Question 5: D

Question 6: B

Question 7: C

Question 8: C

Question 9: D

Question 10: A

Question 11: D

Question 12: E

Question 13: E

Question 14: A

Question 15: C

Question 16: D

Question 17: B

Question 18: B

Question 19: B

Question 20: D

Question 21: A

Question 22: E

Question 23: B

Question 24: A

Question 25: C

Question 26: E

Question 27: D

Question 28: B

Question 76: D

Question 77: E

Question 78: A

Question 79: C

Question 80: E

Question 81: A

Question 82: E

Question 83: B

Question 84: D

Question 85: C

Question 86: E

Question 87: E

Question 88: D

Question 89: E

Question 90: A

Question 91: B

Question 92: A

Free-Response Scoring Guidelines

Question 1

Let R be the shaded region in the first quadrant enclosed by the y-axis and the graphs of $y = 1 - x^3$ and $y = \sin(x^2)$, as shown in the figure above.
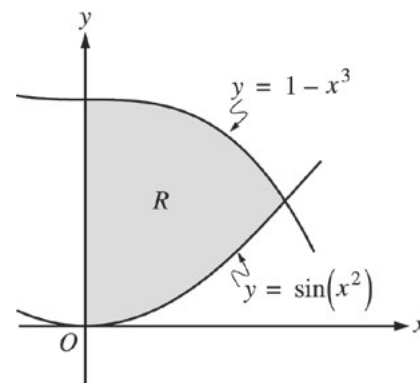
(a) Find the area of R.

(b) A horizontal line, $y = k$, is drawn through the point where the graphs of $y = 1 - x^3$ and $y = \sin(x^2)$ intersect. Find k and determine whether this line divides R into two regions of equal area. Show the work that leads to your conclusion.

(c) Find the volume of the solid generated when R is revolved about the line $y = -3$.

The graphs of $y = 1 - x^3$ and $y = \sin(x^2)$ intersect in the first quadrant at the point $(A, B) = (0.764972, 0.552352)$.

(a) Area $= \int_0^A \left(1 - x^3 - \sin(x^2)\right) dx$

$= 0.533$ (or 0.534)

$2 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $k = B = 0.552352$

$\int_0^A \left(1 - x^3 - k\right) dx = 0.257$ (or 0.256)

$\int_0^A \left(k - \sin(x^2)\right) dx = 0.277$ (or 0.276)

The two regions have unequal areas.

$3 : \begin{cases} 1 : \text{integral(s) with } k \text{ value} \\ 1 : \text{value(s) of integral(s)} \\ 1 : \text{conclusion tied to part (a)} \\ \quad \text{or comparison of two integrals} \end{cases}$

Note: Stating k value only does not earn a point.

(c) Volume $= \pi \int_0^A \left(\left(1 - x^3 + 3\right)^2 - \left(\sin(x^2) + 3\right)^2 \right) dx$

$= 11.841$ (or 11.840)

$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

Question 2

The penguin population on an island is modeled by a differentiable function P of time t, where $P(t)$ is the number of penguins and t is measured in years, for $0 \le t \le 40$. There are 100,000 penguins on the island at time $t = 0$. The birth rate for the penguins on the island is modeled by

$$B(t) = 1000e^{0.06t} \text{ penguins per year}$$

and the death rate for the penguins on the island is modeled by

$$D(t) = 250e^{0.1t} \text{ penguins per year.}$$

(a) What is the rate of change of the penguin population on the island at time $t = 0$?

(b) To the nearest whole number, what is the penguin population on the island at time $t = 40$?

(c) To the nearest whole number, what is the average rate of change of the penguin population on the island for $0 \le t \le 40$?

(d) To the nearest whole number, find the absolute minimum penguin population and the absolute maximum penguin population on the island for $0 \le t \le 40$. Show the analysis that leads to your answers.

(a) $P'(0) = B(0) - D(0) = 1000 - 250 = 750$ penguins per year

1 : answer

(b) $P(40) = 100000 + \displaystyle\int_0^{40} (B(t) - D(t))\, dt$

$\qquad = 100000 + 33057.56459$

There are 133,058 penguins on the island.

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) $\dfrac{1}{40} \displaystyle\int_0^{40} (B(t) - D(t))\, dt = 826.439$

OR

$\dfrac{P(40) - P(0)}{40 - 0} = \dfrac{133058 - 100000}{40} = 826.45$

The average rate of change is 826 penguins per year.

1 : answer

(d) $B(t) - D(t) = 0$

$1000e^{0.06t} = 250e^{0.1t} \Rightarrow t = A = \dfrac{\ln 4}{0.04} = 34.657359$

The absolute minimum and absolute maximum occur at a critical point or at an endpoint.

$P(0) = 100000$

$P(A) = 100000 + \displaystyle\int_0^{A} (B(t) - D(t))\, dt = 139166.667$

$P(40) = 133058$

The minimum population is 100,000 and the maximum population is 139,167 penguins.

$4 : \begin{cases} 1 : B(t) - D(t) = 0 \\ 1 : \text{solves for } t \\ 1 : \text{minimum value} \\ 1 : \text{maximum value} \end{cases}$

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \leq t \leq 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\, dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

(a) An equation of the tangent line is $y = 0.5(t - 30) + 125$.
$W(32) \approx 0.5(32 - 30) + 125 = 126$

$1 : $ answer

(b) $\int_0^{30} W'(t)\, dt \approx (10)(0.6) + (12)(0.7) + (8)(1.0) = 22.4$

$W(0) = W(30) - \int_0^{30} W'(t)\, dt = 125 - 22.4 = 102.6$

$3 : \begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{answer} \end{cases}$

(c) W' is differentiable $\Rightarrow$ W' is continuous.

$W'(30) = 0.5 < 0.7 < 1.0 = W'(22)$

By the Intermediate Value Theorem, there must be at least one time t, $22 \leq t \leq 30$, such that $W'(t) = 0.7$.

$2 : $ explanation

(d) $\dfrac{dA}{dt} = (0.3)\dfrac{2}{3}W^{-1/3} \cdot \dfrac{dW}{dt} = \dfrac{0.2}{\sqrt[3]{W}} \cdot \dfrac{dW}{dt}$

$\dfrac{dA}{dt}\Big|_{t=30} = \dfrac{0.2}{\sqrt[3]{125}} \cdot 0.5 = 0.02$

$3 : \begin{cases} 2 : \dfrac{dA}{dt} \\ 1 : \text{answer} \end{cases}$

Question 4

Let f be the function given by $f(x) = (x^2 - 2x - 1)e^x$.

(a) Find $\lim\limits_{x \to \infty} f(x)$ and $\lim\limits_{x \to -\infty} f(x)$.

(b) Find the intervals on which f is increasing. Show the analysis that leads to your answer.

(c) Find the intervals on which the graph of f is concave down. Show the analysis that leads to your answer.

(a) $\lim\limits_{x \to \infty} f(x) = \infty$ or does not exist

  $\lim\limits_{x \to -\infty} f(x) = 0$

1 : answers

(b) $f'(x) = (2x - 2)e^x + (x^2 - 2x - 1)e^x$
  $\quad = (x^2 - 3)e^x$

  $f'(x) = 0$ when $x = -\sqrt{3}$, $x = \sqrt{3}$

  $f'(x) > 0$ for $-\infty < x < -\sqrt{3}$ and $\sqrt{3} < x < \infty$.

  f is increasing on the intervals $-\infty < x \le -\sqrt{3}$ and $\sqrt{3} \le x < \infty$.

$4 : \begin{cases} 2 : f'(x) \\ 1 : \text{analysis} \\ 1 : \text{intervals} \end{cases}$

(c) $f''(x) = 2xe^x + (x^2 - 3)e^x$
  $\quad = (x^2 + 2x - 3)e^x = (x + 3)(x - 1)e^x$

  $f''(x) < 0$ for $-3 < x < 1$

  The graph of f is concave down on the interval $-3 < x < 1$.

$4 : \begin{cases} 2 : f''(x) \\ 1 : \text{analysis} \\ 1 : \text{interval} \end{cases}$

Question 5

The graph of the continuous function f, consisting of three line segments and a semicircle, is shown above. Let g be the function given by

$$g(x) = \int_{-2}^{x} f(t)\, dt.$$

(a) Find $g(-6)$ and $g(3)$.

(b) Find $g'(0)$.

(c) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a horizontal tangent. Determine whether g has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

(d) Find all values of x on the open interval $-6 < x < 3$ for which the graph of g has a point of inflection. Explain your reasoning.
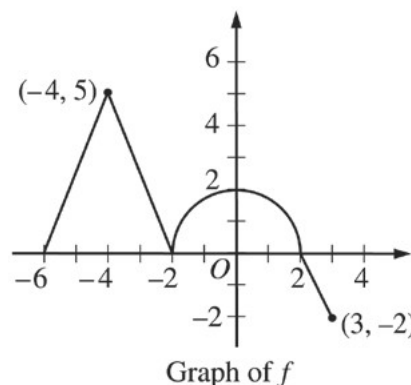
Graph of f

(a) $g(-6) = \int_{-2}^{-6} f(t)\, dt = -\int_{-6}^{-2} f(t)\, dt = -\frac{1}{2} \cdot 4 \cdot 5 = -10$

$g(3) = \int_{-2}^{3} f(t)\, dt = \frac{1}{2}\pi \cdot 2^2 - \frac{1}{2} \cdot 1 \cdot 2 = 2\pi - 1$

$2 : \begin{cases} 1 : g(-6) \\ 1 : g(3) \end{cases}$

(b) $g'(0) = f(0) = 2$

$1 : g'(0)$

(c) The graph of g has a horizontal tangent at $x = -2$ and $x = 2$ where $g'(x) = f(x) = 0$.

The graph of g has neither a local maximum nor a local minimum at $x = -2$ because $g'(x) = f(x)$ does not change sign at $x = -2$.

The graph of g has a local maximum at $x = 2$ because $g'(x) = f(x)$ changes sign from positive to negative at $x = 2$.

$3 : \begin{cases} 1 : \text{horizontal tangent at } x = -2 \\ \quad\quad \text{and } x = 2 \\ 2 : \text{answers with justifications} \end{cases}$

(d) The graph of g has a point of inflection at $x = -4$, $x = -2$, and $x = 0$.
$g'(x) = f(x)$ changes from increasing to decreasing at $x = -4$ and $x = 0$, and changes from decreasing to increasing at $x = -2$.

OR

$g''(x) = f'(x)$ changes from positive to negative at $x = -4$ and $x = 0$, and changes from negative to positive at $x = -2$.

$3 : \begin{cases} 2 : \text{values of } x \\ 1 : \text{explanation} \end{cases}$

Question 6

Let f be a function with $f(2) = -8$ such that for all points (x, y) on the graph of f, the slope is given by $\dfrac{3x^2}{y}$.

(a) Write an equation of the line tangent to the graph of f at the point where $x = 2$ and use it to approximate $f(1.8)$.

(b) Find an expression for $y = f(x)$ by solving the differential equation $\dfrac{dy}{dx} = \dfrac{3x^2}{y}$ with the initial condition $f(2) = -8$.

(a) Slope $= \dfrac{(3)(4)}{-8} = -\dfrac{3}{2}$

An equation for the tangent line is $y = -\dfrac{3}{2}(x - 2) - 8$.

$f(1.8) \approx -\dfrac{3}{2}(1.8 - 2) - 8 = -7.7$

$3: \begin{cases} 1 : \text{slope} \\ 1 : \text{tangent line equation} \\ 1 : \text{approximation} \end{cases}$

(b) $\displaystyle \int y \, dy = \int 3x^2 \, dx$

$\dfrac{1}{2} y^2 = x^3 + C$

$\dfrac{1}{2}(-8)^2 = 2^3 + C \implies C = 24$

$y^2 = 2(x^3 + 24) = 2x^3 + 48$

$y = -\sqrt{2x^3 + 48}$

Note: This solution is valid for $x > -\sqrt[3]{24}$.

$6: \begin{cases} 1 : \text{separation of variables} \\ 2 : \text{antiderivatives} \\ 1 : \text{constant of integration} \\ 1 : \text{uses initial condition} \\ 1 : \text{solves for } y \end{cases}$

Note: max $3/6$ [1-2-0-0-0] if no constant of integration

Note: $0/6$ if no separation of variables

Scoring Worksheet

The following provides a worksheet and conversion table
used for calculating a composite score of the exam.

2013 AP Calculus AB Scoring Worksheet

Section I: Multiple Choice

_____ × 1.2000 = _____
Number Correct   Weighted Section I Score
(out of 45)   (Do not round)

Section II: Free Response

Question 1   _____ × 1.0000 = _____
                      (out of 9)   (Do not round)

Question 2   _____ × 1.0000 = _____
                      (out of 9)   (Do not round)

Question 3   _____ × 1.0000 = _____
                      (out of 9)   (Do not round)

Question 4   _____ × 1.0000 = _____
                      (out of 9)   (Do not round)

Question 5   _____ × 1.0000 = _____
                      (out of 9)   (Do not round)

Question 6   _____ × 1.0000 = _____
                      (out of 9)   (Do not round)

                                        Sum = _____
                                                   Weighted
                                                   Section II
                                                   Score
                                                (Do not round)

Composite Score

_____ + _____ = _____
Weighted   Weighted   Composite Score
Section I Score   Section II Score   (Round to nearest
                                               whole number)

AP Score Conversion Chart
Calculus AB

Composite Score Range	AP Score
69-108	5
55-68	4
44-54	3
36-43	2
0-35	1

AP Calculus AB

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